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Sample Topic

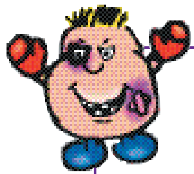
from the

Y10 Intermediate GCSE text

You have permission to print this topic and try it with your students.

This document contains the complete topic (p01–38) and answers (p39-41)

Teachers' notes, worksheets, revision and assessment for this topic can be downloaded and printed off in the Y10 Ruff Resource Sample document.



Ruff

ENJOYMENT

The 'Maths is …' Jugglers

Knowledge Skills Understanding

RUFF GUIDE : Grades E-B

Y10

Topic 6

Coordinates, Shapes and Transformations

CONTENTS

A list of items at Grade B in the Ruff Guides can be downloaded from the website.
www.mathsisjugglers.co.uk

For ALL numerical calculations:

- mental arithmetic or pencil-and-paper techniques should be used wherever possible
- calculators should only be used if the arithmetic is very complex

Key: means that a calculator is essential for one or more pieces in this section.

CONTENTS

of the

Intermediate GCSE Course

THE RUFF GUIDE *Part 1* (Y10)

ISBN-10: 1-874428-88-3
ISBN-13: 978-1-874428-88-6

THE RUFF GUIDE *Part 2* (Y11)

ISBN-10: 1-874428-89-1
ISBN-13: 978-1-874428-89-3

The emphasis here is on non-calculator skills, with a particular stress on mental agility. Many of the sections within the topics open with items that can be used as mental/oral starters and the techniques taught/reviewed here should be repeated regularly over the weeks following their introduction

The course should start with Topic 1. This contains the number techniques that will be assumed thereafter throughout the course.
The rest of the topics are independent and can be done in any order.
Any techniques required within a topic that are taught elsewhere, will be repeated at the point where they are required. *Ruff*

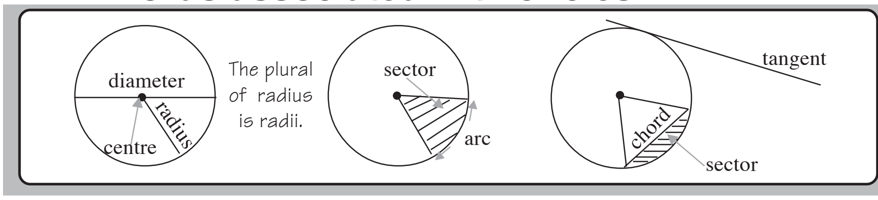
Coordinates, Shapes and Transformations

Section 1: Circles

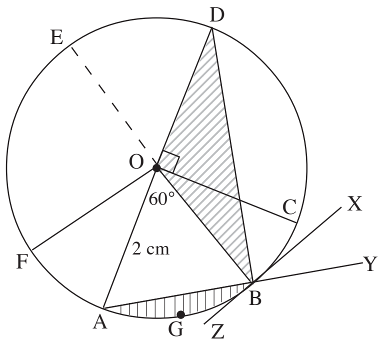
In this section you will review the terminology and properties of circles.

DEVELOPMENT

D1.1: Words associated with circles



Fill in the gaps:



1. …… and …… are diameters
2. …… and …… are chords
3. …… is a tangent to the circle

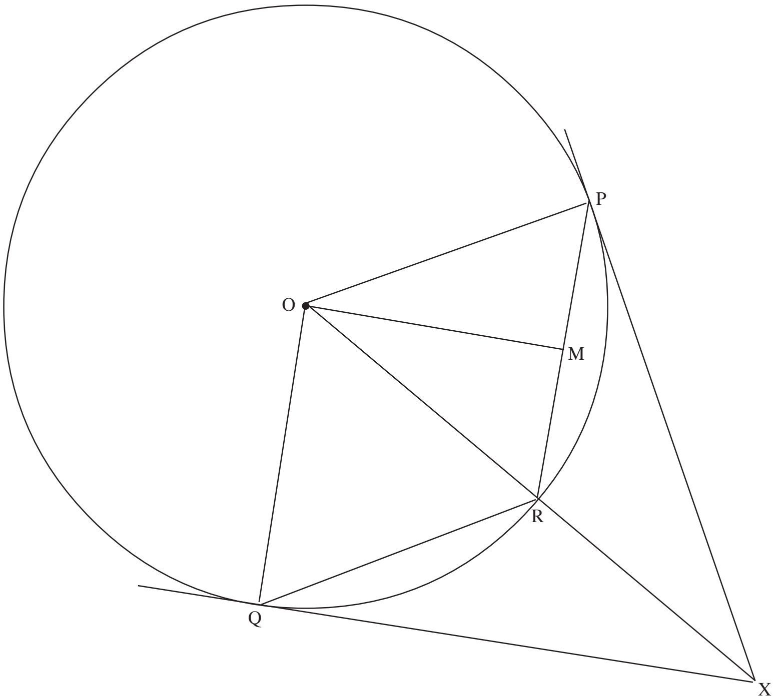
State whether true (T) or false (F):

4. OE is a radius.
5. Sector COE is $^1/_4$ of the area of the circle.
6. Sector COD is $^1/_4$ of the area of the circle.
7. Arc CD is smaller than arc AF.
8. Arc CD is $^1/_4$ of the circumference.
9. Triangle AOB is equilateral.
10. Chord AB is 2 cm long.
11. Sector AOB is $^1/_6$ of the area of the circle.
12. A segment has one straight edge and one curved edge.
13. A sector has two straight edges and two curved edges.
14. The two straight edges of a sector are radii.
15. For a particular circle, all radii are equal in length.
16. The centre of a circle is the midpoint of its diameter.
17. A diameter of a circle is twice as long as the radius of the circle.
18. Chord BD is more than 4 cm long.
19. Segment BCD is shaded.
20. Segment AGB is smaller than segment BCD
21. ABY is a tangent to the circle.

• *Check your answers.*

D1.2: Properties of circles

ruler



Measure the length of…

1. the radius of the circle
2. the length of a tangent from X to the point where it touches the circle
3. the length of the chord PR
4. the length of the line OM, where M is the midpoint of the chord PR

State whether true (T) or false (F):

5. The two tangents from X are equal in length
6. The radius at P is perpendicular to the tangent at P
7. Chord PR is longer than the radius of the circle.
8. OM is is perpendicular to PR, where M is the midpoint of the chord PR.
9. ΔOQR is congruent to ΔOPR
10. ΔOQX is right-angled.

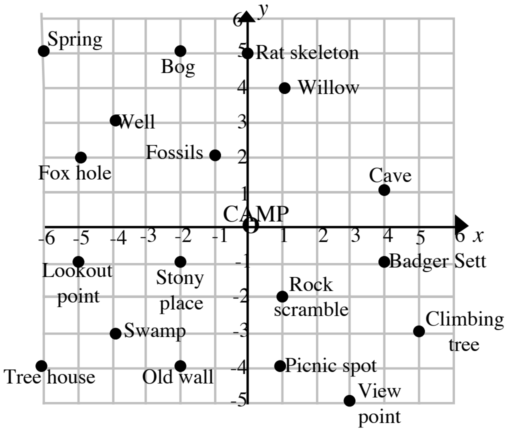
• *Check your answers.*

Section 2: Coordinates and shapes

In this section you will:
- review coordinates in all four quadrants
- work with coordinates and shapes

DEVELOPMENT

D2.1: Olly's camp



Olly had built a camp in the forest. He made a plan showing the places he had found.
The coordinates of the place where he found fossils are (–1, 2).

1. What did he find at (–5, 2) ?
2. What did he watch at (4,–1) ?
3. Where did he find the rat skeleton ?
4. *Copy and complete this table of places and coordinates:*

Place	Fossils	Cave	Rat Skeleton	Badger Sett	………..
Coords	(–1,2)	(…, …)	(…, …)	(…, …)	(1,4)

Place	Picnic spot	Bog	………..	Stony place	………..
Coords	(… , …)	(… , …)	(–5, 2)	(… , …)	(3, –5)

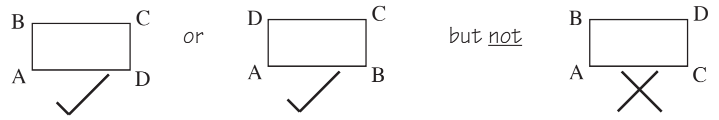
Place	Old wall	………	Tree house	Lookout point
Coords	(… , …)	(–4, –3)	(… , …)	(… , …)

Place	………..	Camp	Climbing tree	Rock Scramble
Coords	(–6, 5)	(… , …)	(… , …)	(… , …)

• Check your answers.

D2.2: Shapes and coordinates

When labelling the vertices of shapes, the labels always go round the shape in order.



On squared paper draw a coordinate grid with values of x from -12 to +12 and values of y from -12 to +12.
Each of the following questions is to be done on this one grid.

1. ABCD is a rectangle. A is (2,4), B is (2,2) and C is (6,2)
   (a) Draw the rectangle on the grid.
   (b) State the coordinates of: (i) D
   (ii) the centre of the rectangle
2. PQR is an isosceles triangle. P is (1,8) and Q is (3,12). PQ = QR
   (a) Draw the triangle on the grid.
   (b) State the coordinates of R
3. LMNP is a parallelogram. L is (6,-3), M is (2,-3) and P is (5,-5)
   (a) Draw the parallelogram.
   (b) State the coordinates of: (i) N
   (ii) the centre of the parallelogram
   (iii) the midpoint of LP
4. WXYZ is a square. W is (-10,3), Z is (-12,6) and Y is (-9,8)
   (a) Draw the square.
   (b) State the coordinates of: (i) X
   (ii) the centre of the square
5. FGHI is a rectangle. F is (8,-2), G is (10,-3), and H is (12,1)
   Find the coordinates of: (i) I
   (ii) the midpoint of FI
   (iii) the centre of the rectangle
6. ABC is a triangle. A is (7,-11), B is (7,-6) and C is (11,-9)
   Find the coordinates of: (i) the midpoint of AB
   (ii) the midpoint of AC
7. KLMN is a parallelogram. K is (-8,-3), L is (-4,1) and M is (-3,-2)
   Find the coordinates of: (i) N
   (ii) the midpoint of KL
   (ii) the midpoint of MN
8. (a) Draw a circle of radius 2 units, with centre at (-10,-9)
   (b) Draw the two tangents which touch the circle at (-8,-9) and (-10,-11)
   (c) State the coordinates of the point where the tangents meet.

• Check your answers.

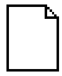
Section 3: Mirror symmetry

In this section you will:

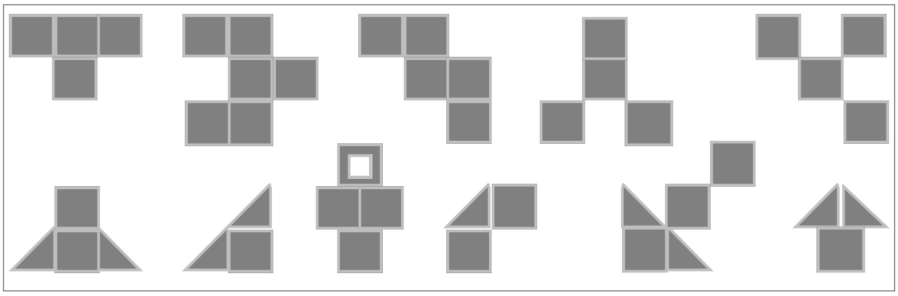
- recognise mirror symmetry
- create symmetrical patterns
- work with 3-D symmetry

DEVELOPMENT

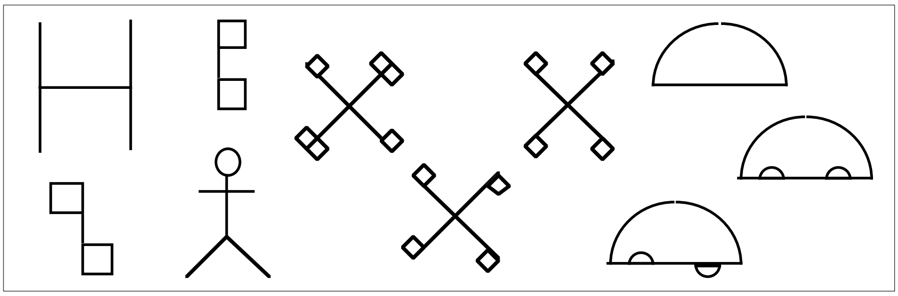
D3.1: Finding lines of symmetry



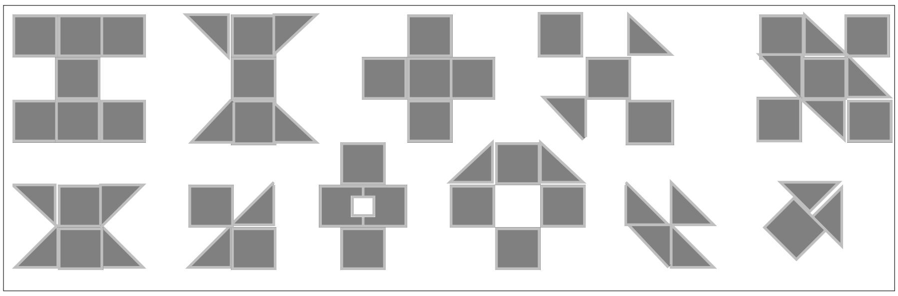
Task 1: *Draw in one line of symmetry for each shape:*



Task 2: *Draw as many lines of symmetry as you can find:*

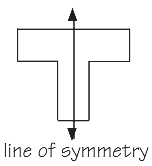


Task 3: *Draw as many lines of symmetry as you can find:*



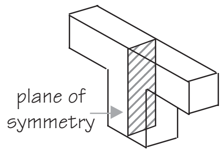
• Check your answers.

D3.2: 3-D symmetry

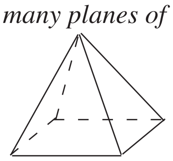


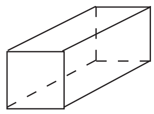
Flat shapes have **lines of symmetry**

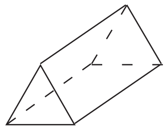
3-D shapes have **planes of symmetry**



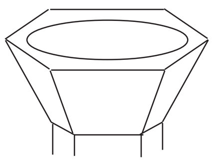
How many planes of symmetry do each of these 3-D shapes have ?

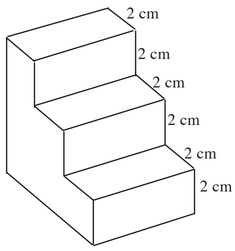
1. 
a square based pyramid

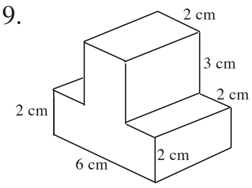
2. 
a cuboid

3. 
an equilateral triangular prism

4. a table with a rectangular top
5. a teacup
6. a three legged stool

7. 
the top of a bath tap

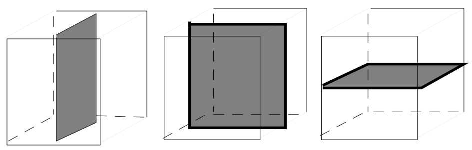
8. 

9. 

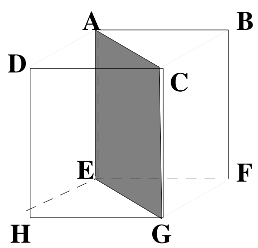
• Check your answers.

EXTENSION

E3.3: *Planes of symmetry of a cube



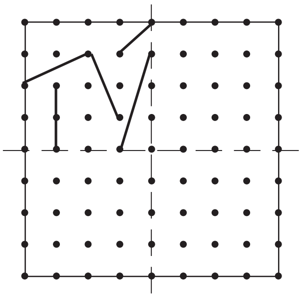
Four of the planes of symmetry of a cube are shown here.
The fourth plane is a diagonal plane of symmetry.
It can be described as ACGE.



There are five further diagonal planes of symmetry. Draw each of these planes and describe them using the letters in the diagram.

• Check your answers.

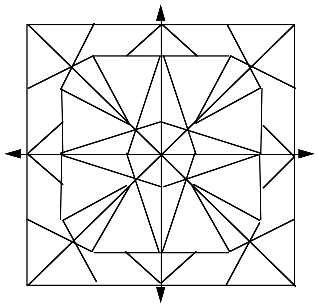
D3.3: Rangoli patterns



Step 1: Reflect the five lines in the top left quadrant in the two (dotted) lines of symmetry.

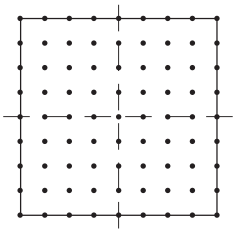
Step 2: Draw the two diagonals of the original, large square. Reflect all the lines in these two diagonals.

You should end up with the Rangoli pattern shown here.



EXTENSIONS

E3.4: *Rangoli challenge



Make Rangoli pattern(s) using five or six lines of your own choice.

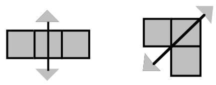
Rangoli patterns are patterns that Hindu families make to decorate their homes for the Dirwali festival.

You can start with any number of lines – but too many lines make a too-fussy pattern

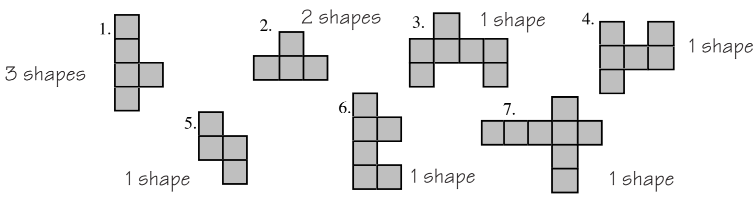
E3.5: *Add a square

Adding  to

you can make two different symmetric shapes



Add a square to each of these shapes.
The statement 'n shapes' gives the number of different shapes you should make.
Shade the square you have added in a different way to the other squares.
Draw in the lines of symmetry.



Section 4: Rotational symmetry

In this section you will:
- review rotational symmetry
- find the symmetries of polygons

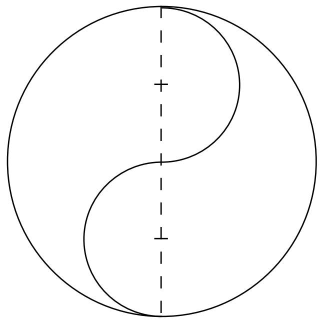
DEVELOPMENT

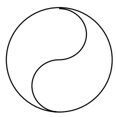
D4.1: Circle Patterns

pencil compasses eraser tracing paper

Task 1: Circle Pattern A

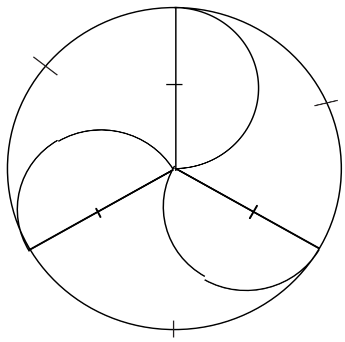
- Draw a circle with radius 4 cm
- Draw a diameter
- Divide the diameter into four equal parts
- Draw the two semi-circles as shown
- Rub out the diameter
  to get Circle Pattern A

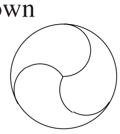




Task 2: Circle Pattern B

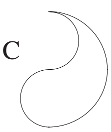
- Draw a circle with radius 4 cm
- Keep the compass distance 4 cm
  and use it to mark six points at
  equal distances round the circle
- Draw radii from three of the points
  (as shown)
- Draw semi-circles as shown
- Rub out the radii
  to get Circle Pattern B

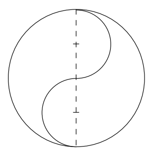




Task 3: Circle Pattern C

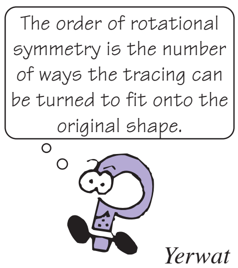
- Draw another Circle Pattern A
- Rub out the diameter and half of
  the large circle to get Circle Pattern C





Task 4: Trace each circle pattern.
Work out the order of rotational symmetry of each.
Put your results into a table like this:

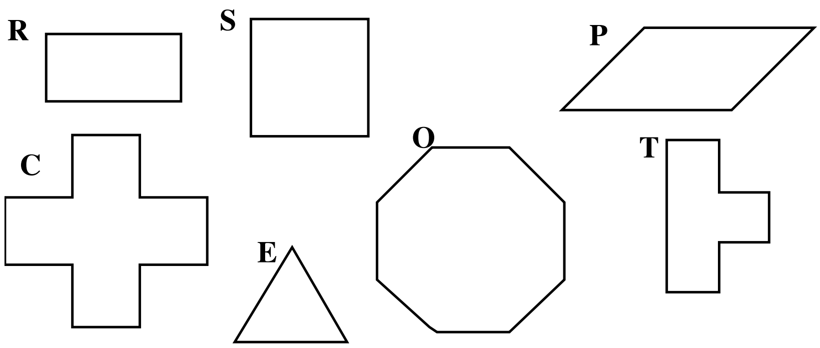
Circle Pattern	A	B	C
Order of rotational symmetry			



• Check your answers.

Yerwat

D4.2: Rotational symmetry



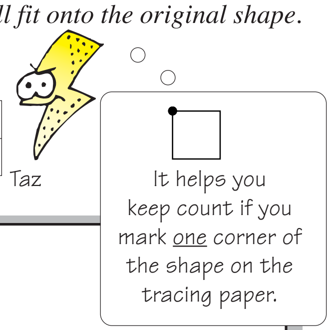
Task 1: *Trace one shape at a time.*

Count how many different ways the tracing will fit onto the original shape.

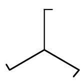
Do not turn the tracing paper over.

Put your results onto a table like this:

Shape	R	S	P	C	E	O	T
No. of ways							



A tracing of this shape will fit on the shape in 3 different ways.
It has <u>rotational symmetry of order 3</u>.



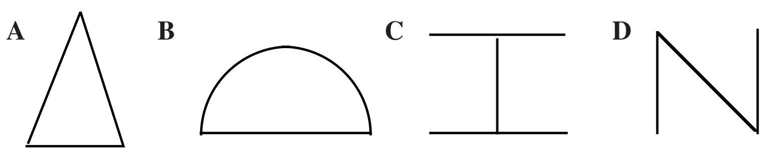
Every shape has order of symmetry of at least 1.
[It will fit onto its own shape at least once]

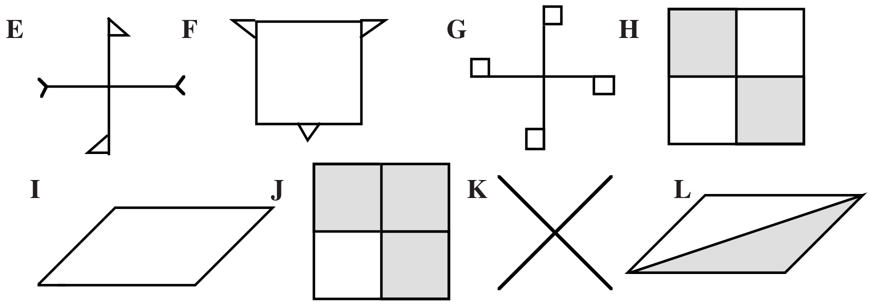
We say that a shape has rotational symmetry if its order is more than 1.

Task 2: *For each of the following 12 shapes, find:*

- *the order of rotational symmetry*
- *the number of lines of symmetry*

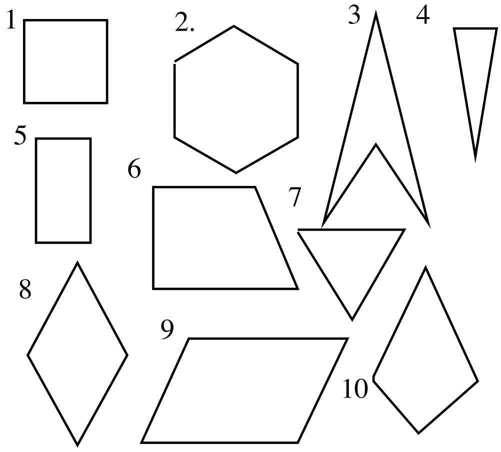
Put your results into a table.





• *Check your answers*

D4.3: Polygon symmetry



K Kite
R Rectangle
E Equilateral triangle
T Trapezium
P Parallelogram
D Diamond (rhombus)
H Hexagon
A Arrowhead
I Isosceles triangle
S Square

Match each shape to the letter attached to its name and fill in this table:

Shape	1	2	3	4	5	6	7	8	9	10
Letter of name										
No. of lines of symmetry										
Order of symmetry										

• *Check your answers..*

Section 5: Transformations

In this section you will meet four different kinds of transformations.

DEVELOPMENT

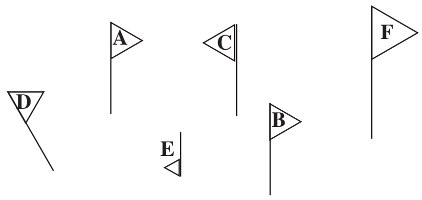
D5.1: Which transformation could it be ?

Transformations are ways of moving shapes from one position to another.

A **translation** slides the shape from one position to another.
Flag B is a translation of flag A.

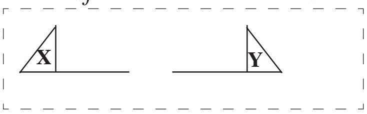
A **reflection** flips the shape over.
Flag C is a reflection of flag A.

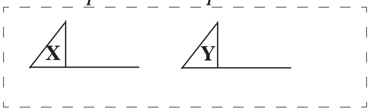
A **rotation** turns the shape around.
Flag D is a rotation of flag A.

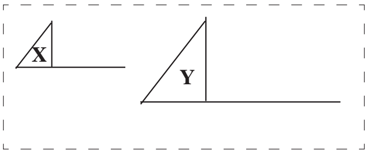


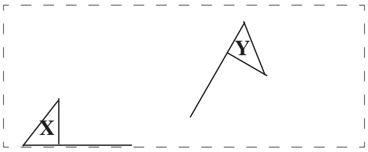
An **enlargement** changes the size and position of the shape.
Flags E and F are enlargements of flag A.

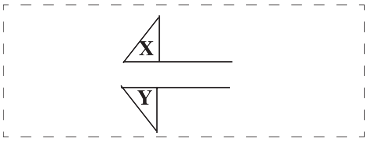
Which transformation could be used to get from shape X to shape Y:

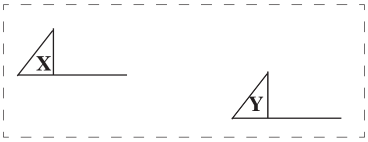
1. 

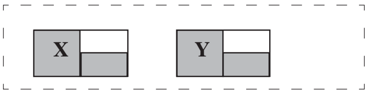
2. 

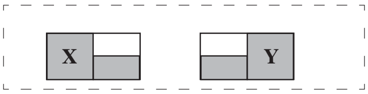
3. 

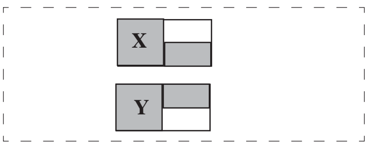
4. 

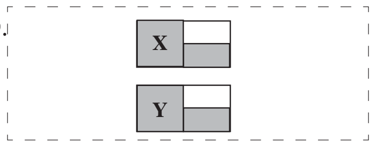
5. 

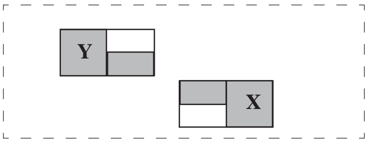
6. 

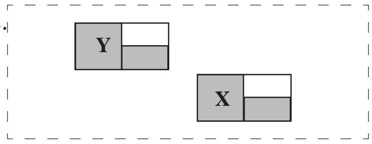
7. 

8. 

9. 

10. 

11. 

12. 

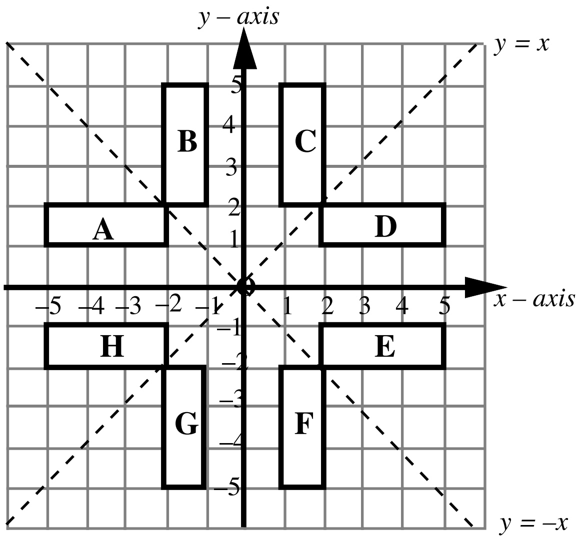
• *Check your answers.*

Section 6: Reflections

In this section you will:
- reflect shapes on a coordinate grid
- review equations of some simple lines

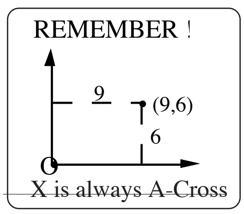
DEVELOPMENT

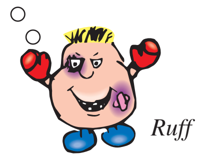
D6.1: Reflections on coordinates



State which rectangle is referred to in each case:

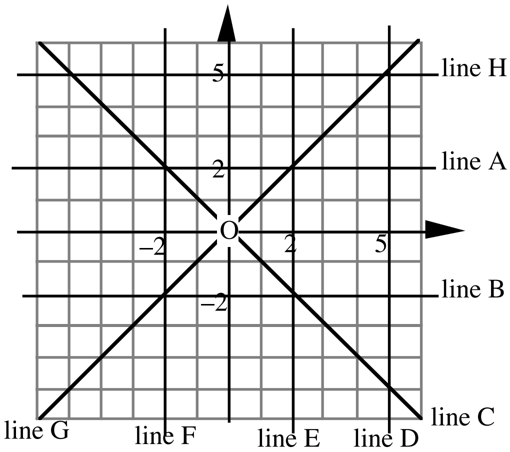
1. … is the rectangle (1,2) (1,5) (2,2) (2,5)
2. … is the rectangle (2,–1) (2,–2) (5,–2) (5,–1)
3. … is the rectangle (–2,–1) (–2,–2) (–5,–2) (–5,–1)
4. … is the rectangle (1,–2) (1,–5) (2,–2) (2,–5)
5. … is the rectangle (–2,1) (–2,2) (–5,1) (–5,2)
6. … is the rectangle (–1,2) (–1,5) (–2,2) (–2,5)
7. … is the reflection of C in the y–axis.
8. … is the reflection of C in the x–axis.
9. … is the reflection of C in the line $y = x$.
10. … is the reflection of E in the line $y = -x$.
11. … is the reflection of E in the x–axis.
12. … is the reflection of E in the y–axis.
13. … is the reflection of E in the line $y = x$.





• *Check your answers.*

D6.2: Meeting some simple lines again



1. On one of these lines the x–coordinate of every point is 2.
   This line has the equation $x = 2$.
   Which line is $x = 2$?
2. On one of these lines the y–coordinate of every point is –2.
   This line has the equation $y = –2$.
   Which line is $y = –2$?
3. On one of these lines the y–coordinate of every point is equal to the x–coordinate.
   This line has the equation $y = x$.
   Which line is $y = x$?
4. *Match each line with its correct equation:*

Line A
Line B
Line C
Line D
Line E
Line F
Line G
Line H

$x = 2$	$x = –2$	$x = 5$
$y = 2$	$y = –2$	$y = 5$
$y = x$	$y = –x$	

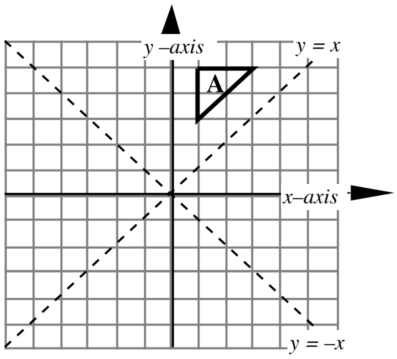
• *Check your answers.*

D6.3: Reflective symmetry

Task 1: *Copy this diagram.*

Draw the images B, C, D, E, F, G, H using the instructions in the table.

Image	Reflection of	Reflected in
B	A	y–axis
C	A	$y = x$
D	B	$y = x$
E	D	$y = -x$
F	D	y–axis
G	F	$y = x$
H	F	x–axis

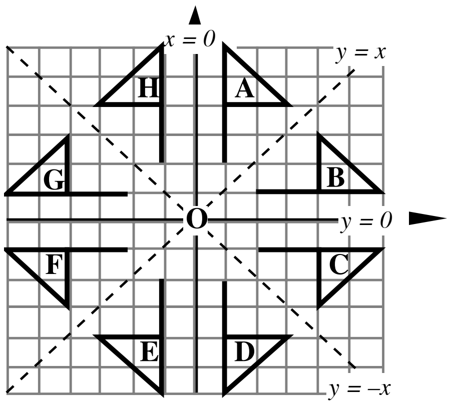


Task 2: Write down the coordinates of the corners of each Δ A, B, C, D, E, F, G, H

• Check your answers.

D6.4: Find the mirror lines

Complete the table of moves and reflections:



Move	Reflection in
A —> B	
A —> D	
H —> G	
B —>	$x = 0$
E —>	$y = 0$
F —> C	
A —>	$y = 0$
H —>	$x = 0$
D —> C	
F —>	$y = -x$

• Check your answers.

EXTENSIONS

E6.5: **Star reflection challenge

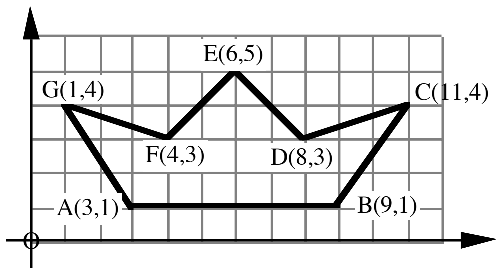
- Draw a set of coordinate axes with values of x from –8 to 8
  and values of y from –8 to 8.
- Draw the triangle with corners (0,0) (2,0) (2,3)
- Draw the triangle with corners (0,0) (6,3) (3,0)
- Draw the mirror line $y = x$ as a dotted line.
- Reflect the two triangles in
  the x–axis
  the y–axis
  and $y = x$ to give a 16 point star.
- Write down the coordinates of each point of the star.

• Check your answers.

E6.6: Rules for coordinate reflections

*Task 1: Reflection in the *x*–axis

1. Copy this diagram.
   Draw the reflection
   of the crown in the x–axis.



2. *Complete this table:*

Point letter	A	B	C	D	E	F	G
Coordinates of point	(3,1)						
Coordinates of image							

3. What happens to the x–coordinate of each point under this reflection ?
4. What happens to the y–coordinate of each point under this reflection ?
5. A triangle has vertices (corners) P(1,2) Q(2,4) R(1,6)
   Imagine that this triangle is reflected in the x–axis. (Do NOT reflect it yet.)
   Complete this statement:
   “"I predict that the images of P, Q, R will be ……”

6. Draw ΔPQR and its reflection. Were your predictions correct ?

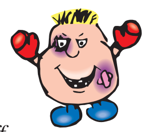
7. Under reflection in the *x*–axis

(3,1) —> (3, –1)

Read this as (3,1) becomes (3,–1)

Complete these, for reflection in the x–axis:

(5,7) —>
(2,–3) —>
(1,4) —>
(–3,–4) —>



Ruff

Read this as *(x,y)* becomes*(x,–y)*

8. Under reflection in the *y*–axis we say *(x,y)* —> *(x, –y)*
Explain what this tells you about what happens to the coordinates.

*Task 2: Reflection in the *y*–axis

9. Draw the crown you were given in Task 1.
Draw its reflection in the *y*–axis.

10. Make a table showing the coordinates of each point and its reflection.

11. Draw a triangle of your own and label its vertices (corners) with their coordinates.
Imagine your triangle reflected in the *y*–axis.
Predict what the coordinates will be of the corners of the reflected triangle.
Write down your predictions.
Check your predictions. Say whether you were correct.

12. *Complete these for reflections in the y–axis*:

(3,2) —>
(4,–1) —>
(–2,5) —>
—> (–3,4)
—> (3,–2)

13. *Complete this statement:*
"Under reflection in the *y*–axis (*x*,*y*) —> ( )
This is the coordinate rule for reflection in the y–axis.

• *Check your answers.*

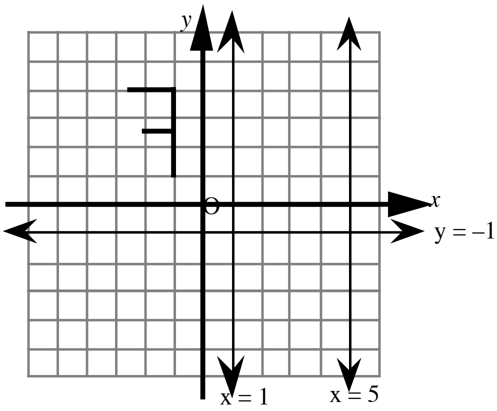
**Task 3: Reflection challenge

Investigate what happens to the coordinates of points under reflection in $y = x$
Explain what you are doing. Show all your working and your results.
Make predictions and check them. What conclusions can you draw ?
Find a coordinate rule for reflection in $y = x$.

• *Show Task 3 to your teacher.*

E6.7: */**Testing hypotheses

You will need to make one copy of this diagram for each of Tasks 1 – 4.



Task 1: Reflect in $x = 1$
Reflect the image in $x = 5$

Task 2: Start again.
Reflect in $x = 5$
Reflect the image in $x = 1$

Task 3: Start again.
Reflect in $x = 1$
Reflect the image in $y = –1$

Task 4: Start again.
Reflect in $y = –1$
Reflect the image in $x = 1$

A hypothesis is a statement that may, or may not, be true.

***Task 5:** Test each of the following hypotheses.
Explain how you know if it is true (or not true).

Hypothesis 1

Reflection in x = 1 followed by reflection in x = 5
is the same as
a translation of 8 —>

Hypothesis 2

Reflection in x = 1 followed by reflection in x = 5
is the same as
reflection in x = 5 followed by reflection in x = 1

Hypothesis 3

Reflection in x = 1 followed by reflection in y = –1
is the same as
reflection in y = –1 followed by reflection in x = 1

Hypothesis 4

After two reflections the shape is the same way round as it was in the beginning.

****Task 6:** Invent some hypotheses of your own. Test them
Explain how you know whether they are true or not.

• *Your teacher will need to mark this.*

Section 7: Rotations

In this section you will:
- recognise and describe rotations of shapes
- rotate shapes according to instructions

DEVELOPMENT

D7.1: Describing rotations

The move from L_1 to L_2 is a **rotation**.

The **angle of rotation** is 90°.

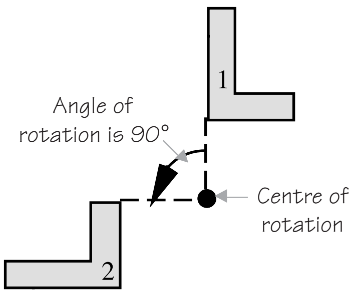
The **centre of rotation** is O.

If the angle of rotation is positive, then the rotation is anti-clockwise. +

If the angle of rotation is negative, then the rotation is clockwise. –

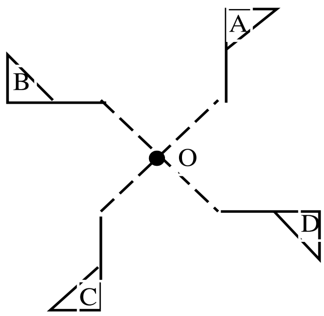
We write : L_1 —> L_2 is a rotation of 90°

We read it as " L_1 to L_2 is a rotation of 90°."



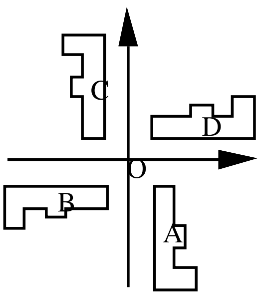
1. *Copy and complete this table for these rotations*:

Move	Angle	Centre
A —> B	+90°	0
B —> A		
D —> B		
D —> A	+90°	
C —> B	–90°	
C —> A		
D —> C		
A —> D		



2. *Copy and complete this table*:

Shortest Move	Rotation about O Angle
A —> B	
A —> C	
B —> C	
A —> D	
D —> B	
C —> A	
C —> D	



• *Check your answers.*

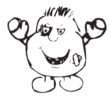
D7.2: Rotating shapes

tracing paper

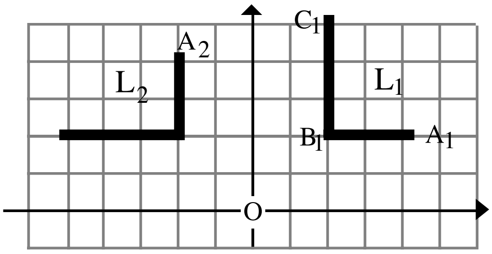
In the diagram below,
$L_1 \longrightarrow L_2$ is a rotation of 90° about O.
The shape L_2 is **the image of the shape** L_1.
The point A_2 is **the image of the point** A_1

Tracing paper is useful when rotating shapes.

Ruff



1.

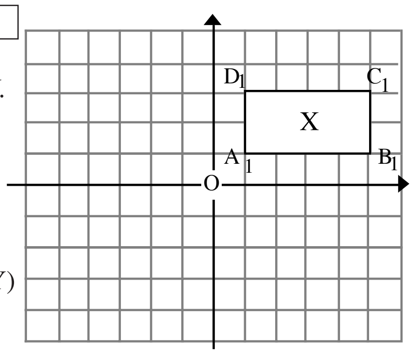


(a) Copy this diagram.

(b) B_2 is the image of B_1 after the rotation. Mark B_2 on the diagram.

(c) C_2 is the image of C_1 after the rotation. Mark C_2 on the diagram.

2. $X \longrightarrow Y$ is a rotation of + 90° about O.



(a) Copy this diagram. Draw the image of the rectangle X. Label the image Y.

(b) The images of the points A_1, B_1, C_1, D_1 are A_2, B_2, C_2, D_2. Mark the points A_2, B_2, C_2, D_2 in the correct positions.

(c) $Y \longrightarrow Z$ is a rotation of –90° about O. Draw the rectangle Z (the image of Y) on the same diagram.

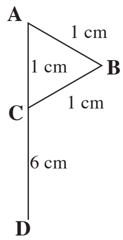
(d) A_3, B_3, C_3, D_3 are the images of A_2, B_2, C_2, D_2 after the rotation. Put them on the diagram.

• *Check your answers.*

D7.3: Rotation about any point

tracing paper

Make 6 accurate copies of the flag.
Do not put the measurements on the copies.
For each copy of the flag, draw its image under the given rotation.
Draw the image in a different colour.



Rotate the flag…

1. … 45° about D	2. … -45° about D
3. … 90° about A	4. … -90° about C
5. … 60° about B	6. … -30° about D

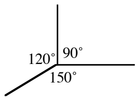
• *Check your answers.*

Section 8: Angle properties

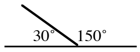
In this section you will review angles at a point, on a line and in triangles.

DEVELOPMENT

D8.1: Angles at a point and on a straight line

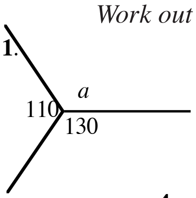


Angles at a point add up to 360°



Angles on a line add up to 180°

Work out the value of each letter used in these angles.

1.


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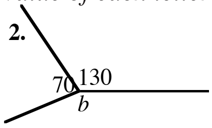

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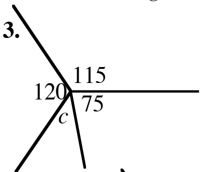

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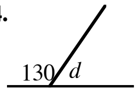

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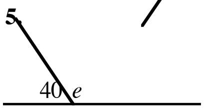

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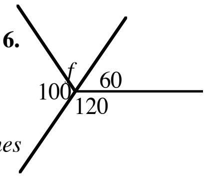
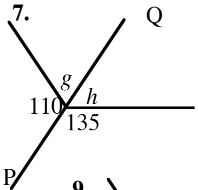


In the following questions, PQ and RS are always straight lines

7.


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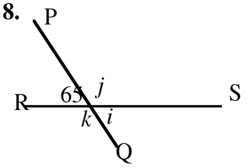

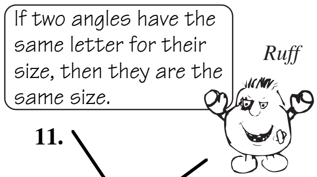

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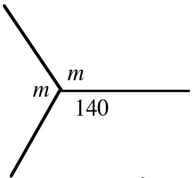

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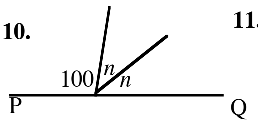

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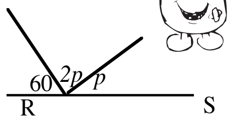

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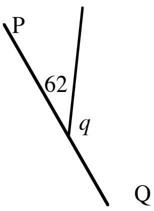

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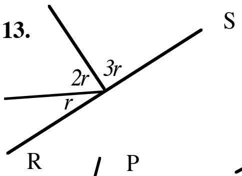

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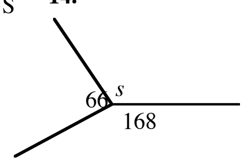

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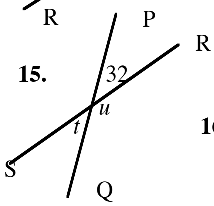
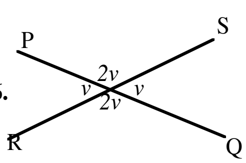


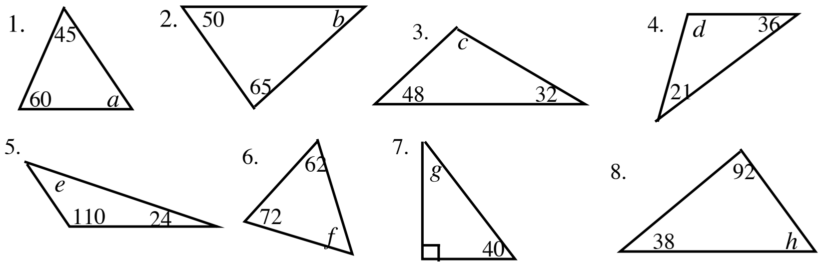
16.


• *Check your answers.*

D8.2: Angles in a triangle

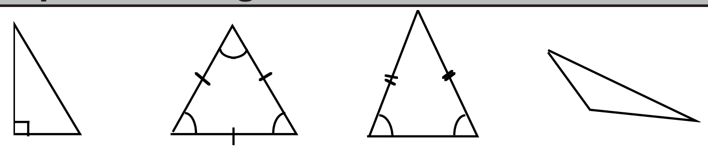
The angles in a triangle add up to 180°

Work out the size of each lettered angle:



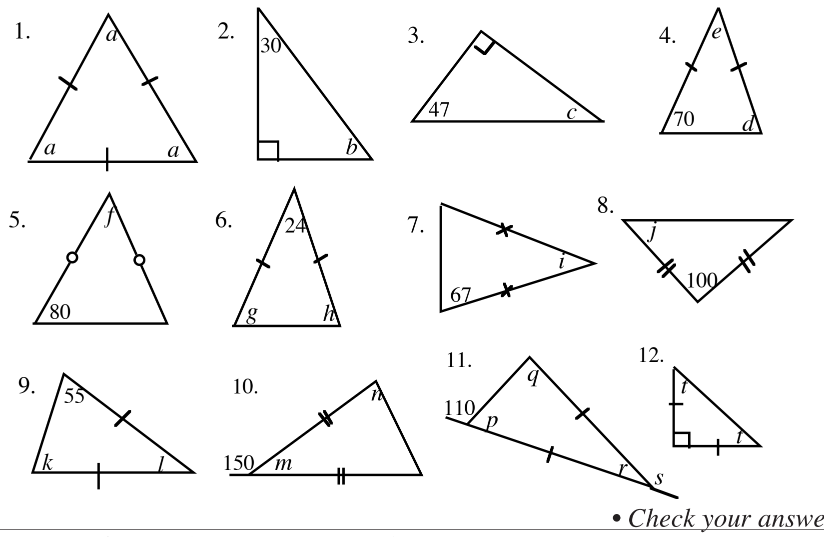
• *Check your answers.*

D8.3: Special triangles



right-angled Δ	equilateral Δ	isosceles Δ	scalene Δ
	3 equal sides	2 equal sides	0 equal sides
	3 equal angles	2 equal angles	0 equal angles

Work out the size of each lettered angle:



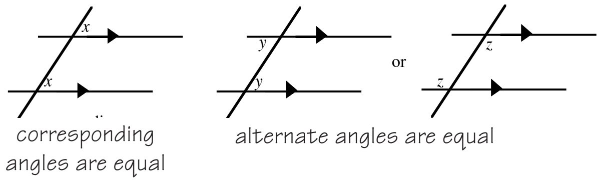
• *Check your answers.*

Section 9: Parallel lines

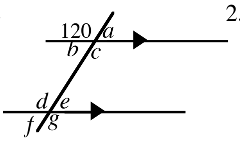
In this section you will work with angles on parallel lines.

DEVELOPMENT

D9.1: Calculating angles on parallel lines



Copy these diagrams. Replace the letters with the correct angle sizes.

1.


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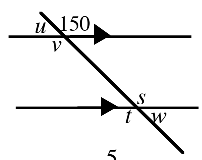

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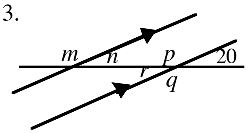

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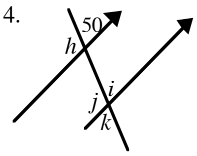

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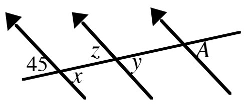
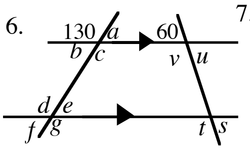


**Work out the angle size represented by each letter:*

6.


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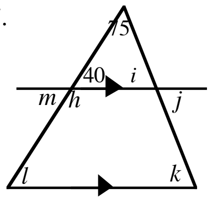

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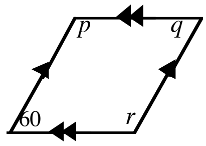
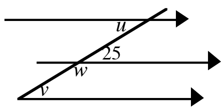


***For each of the questions below, you will need to:*
- *draw a large diagram*
- *put in as many angles as you can find, until you can find the angles you want*
- *work out the size of each lettered angle.*

9.


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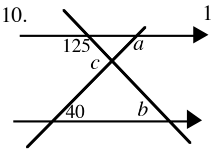

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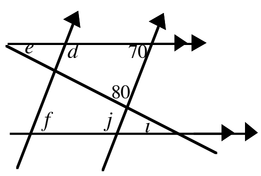
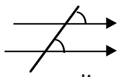
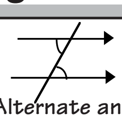


• Check your answers.

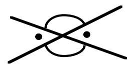
D9.2: Classifying angles on parallel lines



Corresponding angles are equal



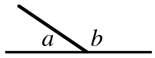
Alternate angles are equal



Vertically opposite angles are equal

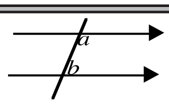
Supplementary angles add up to 180°

a and *b* are supplementary

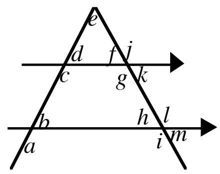


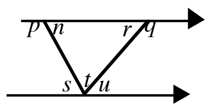
Supplementary interior angles

a + b = 180°



1. *Which angle is :*
   (a) vertically opposite to *h*?
   (b) corresponding to *d*?
   (c) alternate to *l*?
   (d) supplementary to *d*?
   (e) supplementary and interior to *c*?
   (f) corresponding to *g*?
   (g) alternate to *k*?





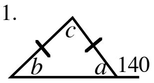
2. *Name two pairs of angles which are:*
   (a) supplementary but not interior
   (b) supplementary interior angles
   (c) alternate angles

• Check your answers.

PRACTICE

P9.3: A mixture of angles

Work out the angle size represented by each letter.

1.


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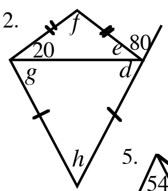

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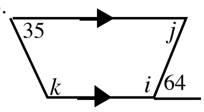

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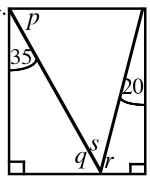

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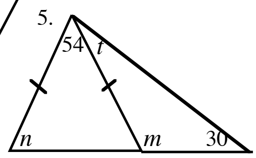

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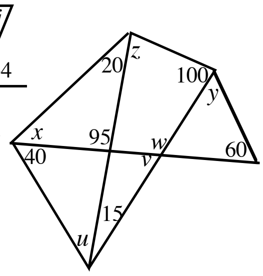
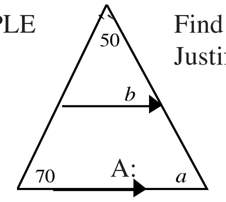


• Check your answers.

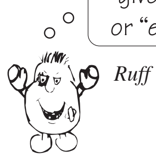
D9.4: Angles and explanations

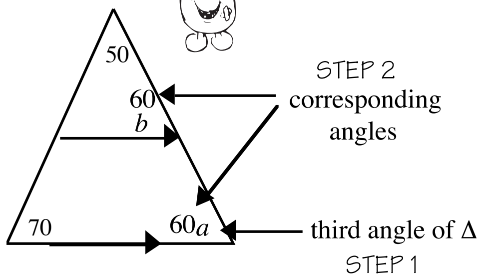
EXAMPLE



Find the values of *a* and *b*.
Justify your values.

"Justify" means "give reasons for" or "explain why"





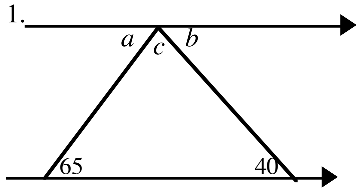
STEP 1, STEP 2… gives the order of working out.

The rest of the explanation on the diagram is the justification for the answers

LARGE CLEAR DIAGRAMS ARE ESSENTIAL !

Find the values of each letter, justifying each value:

1.



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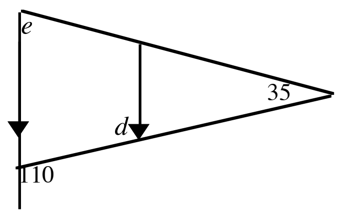

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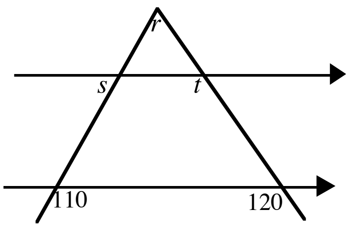

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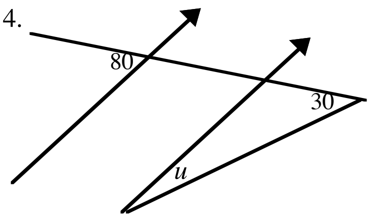

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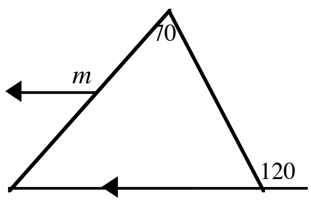
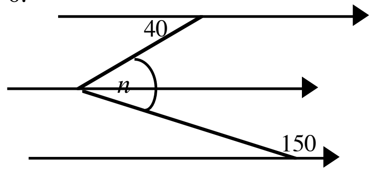


*6.



• Check your answers.

Section 10: Geometric proofs

In this section you will meet several simple geometric proofs.

DEVELOPMENT

D10.1: Some simple proofs

Students need to be able to understand
(and, where possible, reproduce) the following proofs.

1. **To prove:** The sum of the angles in a triangle = 180°

Proof: AB and PQ are parallel lines

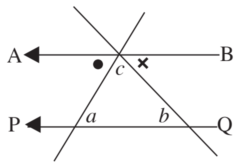
● = a (alternate angles)

× = b (alternate angles)

c + ● + × = 180° (angles on a straight line)

⇒ $a + b + c = 180°$

⇒ the sum of the angles in any triangle = 180°



2. **To prove:** The sum of the angles in a quadrilateral = 360°

Proof: $a + b + c = 180°$ (angles in a triangle)

$d + e + f = 180°$ (angles in a triangle)

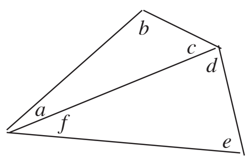
Sum of angles in quadrilateral

$= b + (c + d) + e + (f + a)$

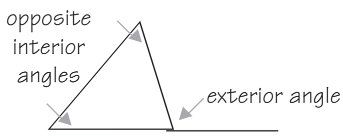
$= a + b + c + d + e + f$

$= 180° + 180°$

$= 360°$



3. **To prove:** The exterior angle of a triangle is equal to the sum of the two opposite interior angles.

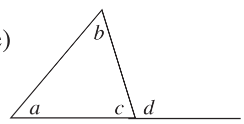


Proof: $a + b + c = 180°$ (angles in a triangle)

$d + c = 180°$ (angles on a straight line)

so, $d = a + b$

⇒ The exterior angle of a triangle is equal to the sum of the two opposite interior angles.

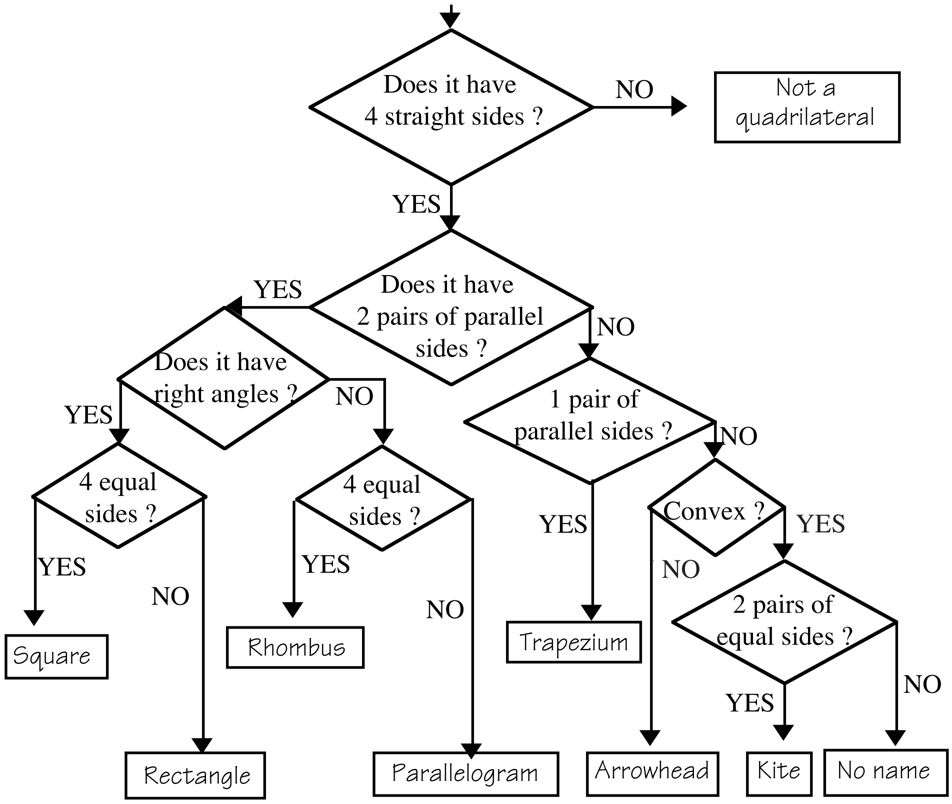


Section 11: Quadrilaterals

In this section you will:
- classify quadrilaterals according to their properties
- learn the properties of quadrilaterals
- use angle and symmetry properties of quadrilaterals to calculate angles.

DEVELOPMENT

D11.1: Classifying quadrilaterals

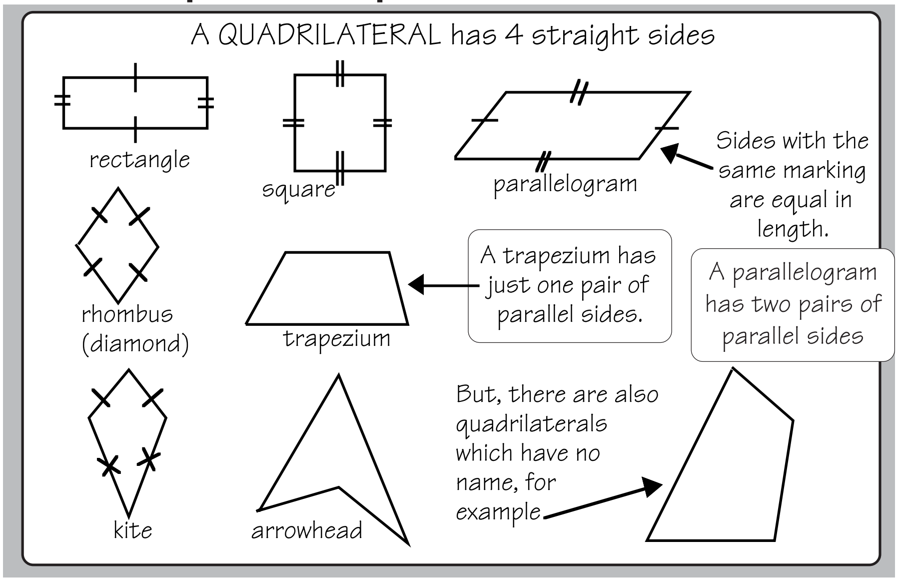


Work out what each shape is:

1. This shape has 2 pairs of parallel sides, all of which are equal, but no right angles.
2. This shape has 4 sides, only two of which are parallel.
3. This shape has no parallel sides and is convex.
4. This shape has no parallel sides, is not convex and has two pairs of equal sides.
5. This shape has 2 pairs of parallel sides, not all equal, and no right angles.

• Check your answers

D11.2: Properties of quadrilaterals



Say whether each of these statements is true or false. If it is false, say why.

1. A square has four right angles.
2. A parallelogram has diagonals of equal length.
3. A kite has two pairs of equal sides.
4. Only a kite has two pairs of equal sides.
5. The diagonals of a rhombus cross at right angles.
6. The diagonals of a rhombus are of equal length.
7. The diagonals of a square cross at right angles.
8. The diagonals of a square are of equal length.
9. The diagonals of a kite cross at right angles.
10. The diagonals of a kite are of equal length.
11. A rectangle is also a parallelogram.
12. A parallelogram is also a rectangle.
13. A trapezium is a special parallelogram.
14. A trapezium can have two right angles.
15. A trapezium can have just one right angle.

Say which TWO quadrilaterals could be defined in each case:

16. A …………… has right angles.
17. A …………… has 2 pairs of parallel sides and diagonals of equal length
18. A …………… has 4 equal sides and perpendicular diagonals.

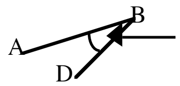
Say which ONE quadrilateral could be defined in each case:

19. A …………… has 2 pairs of parallel sides but no lines of symmetry.
20. A …………… has a pair of unequal parallel sides. • *Check your answers.*

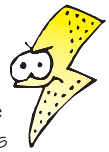
D11.3: Quadrilateral calculations

For each question:

- make a LARGE copy of the diagram;
- put onto your diagram any given information;
- mark any sides of equal length or parallel sides;
- work out the size of the angles asked for;
- show all working out on your diagram;
- write your answers beside the diagram.

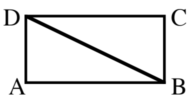


Note: $A\hat{B}D$ is at B, the middle letter, and it lies between AB and BD



Taz

1.

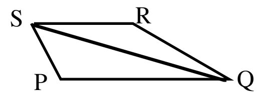


Given: ABCD is a rectangle

$C\hat{D}B = 40^\circ$

Work out: $A\hat{B}D$ $A\hat{C}B$

2.

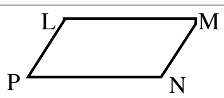


Given: PQRS is a trapezium

$Q\hat{P}S = 110^\circ$ $P\hat{Q}S = 20^\circ$

Work out: $P\hat{S}Q$ $Q\hat{S}R$

3.

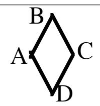


Given: LMNP is a parallelogram

$L\hat{P}N = 75^\circ$ $L\hat{N}M = 80^\circ$

Work out: $L\hat{M}N$ $M\hat{N}P$ $N\hat{L}P$ $M\hat{L}N$

4.

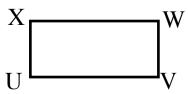


Given: ABCD is a rhombus

$A\hat{B}C = 48^\circ$

Work out: $B\hat{C}A$ $B\hat{D}C$ $D\hat{A}B$

5.

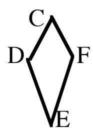


Given: UVWX is a rectangle

M is midpoint of XW $U\hat{M}V = 70^\circ$

Work out: $U\hat{M}X$ $M\hat{V}W$ $M\hat{V}U$

6.

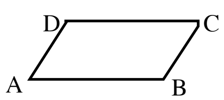


Given: CDEF is a kite

$D\hat{C}F = 140^\circ$ $F\hat{D}E = 60^\circ$

Work out: $F\hat{C}E$ $C\hat{E}D$ $C\hat{F}E$ $C\hat{F}D$

7.

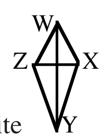


Given: ABCD is a parallelogram

$A\hat{D}B = 65^\circ$ AB = BD

Work out: $A\hat{B}D$ $A\hat{D}C$ $B\hat{C}D$

8.

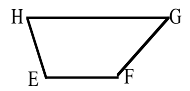


Given: WXYZ is a kite

$X\hat{W}Z = 100^\circ$ $X\hat{Y}Z = 40^\circ$

Work out: $W\hat{X}Z$ $W\hat{X}Y$ $W\hat{Z}Y$

*9.

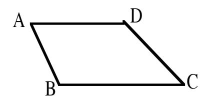


Given: EFGH is a trapezium

$\hat{HEF} = 116°$ EF = EH HF = HG

Work out: $\hat{EFH}$ $\hat{FHG}$ $\hat{EHG}$

*10.

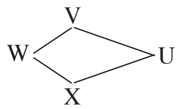


Given: ABCD is a trapezium

AB = AD $\hat{BAD} = 50°$ $\hat{BCD} = 30°$

Work out: $\hat{ADB}$ $\hat{ABC}$ $\hat{DBC}$ $\hat{BDC}$

**11.

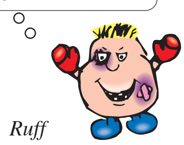


Given: UVWX is a kite

XW = WV = XV $\hat{UWV} = 2 \times \hat{WUV}$

Work out: $\hat{VWX}$ $\hat{XWU}$ $\hat{WUV}$ $\hat{UVX}$

If you can't find the angle directly, put in every angle you can find, until you can see how to get the one you want.

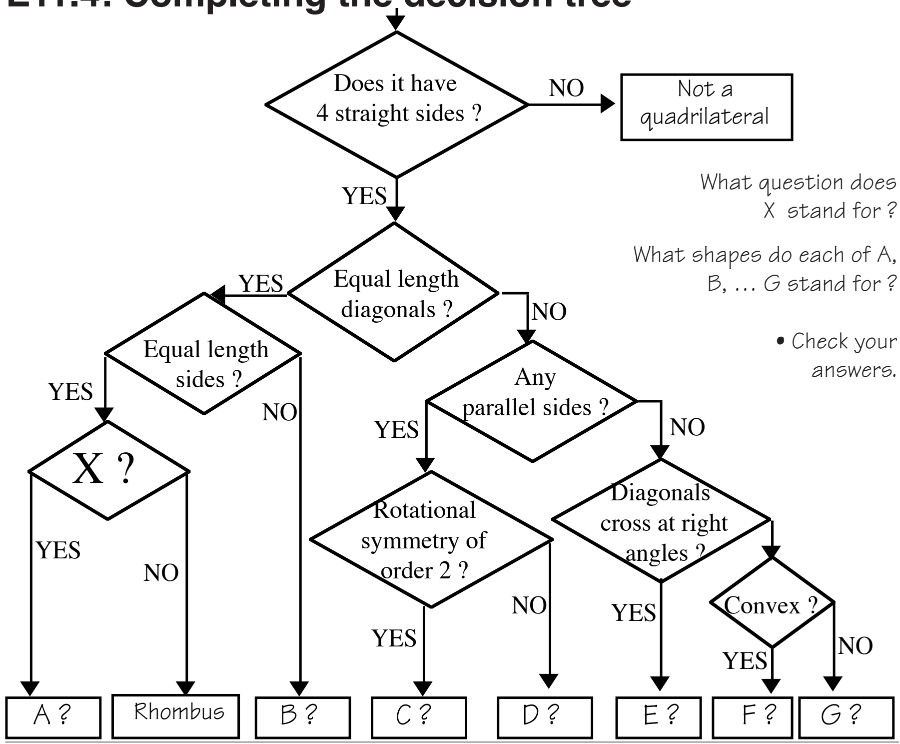


Ruff

• *Check your answers.*

EXTENSION

E11.4: Completing the decision tree



What question does X stand for ?

What shapes do each of A, B, … G stand for ?

• Check your answers.

Ruff : Topic 6: Coordinates, Shapes and Transformations Section 11 page 29

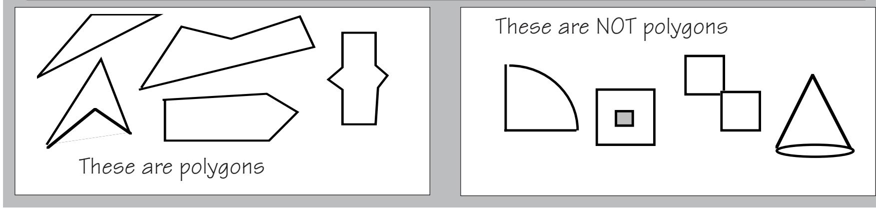
Section 12: Polygons

In this section you will:
- review properties and terminology of polygons
- construct regular polygons

DEVELOPMENT

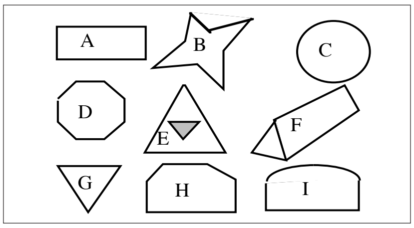
D12.1: Properties of some polygons

A **polygon** is a flat shape with straight sides.



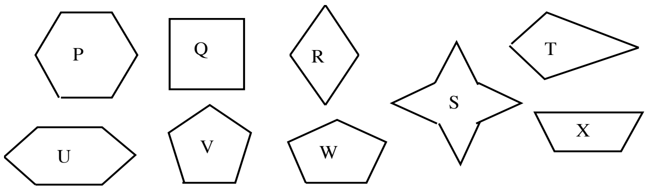
1. Which of these are polygons ?

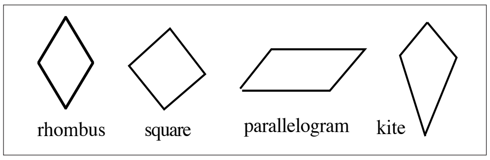
   Give the letters of the polygons.



A **regular polygon** has equal sides and equal angles.

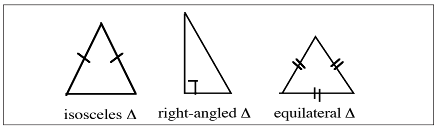
2. Which of these polygons are regular ?





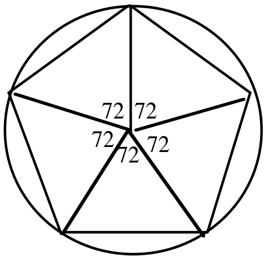
3. What is a regular quadrilateral called ?
4. Which quadrilateral has four equal sides but is <u>not</u> regular ?

5. What is a regular triangle called .



• *Check your answers.*

D12.2: Constructing regular polygons

1. 

A **regular pentagon** is made from 5 congruent (identical) triangles.

(a) What kind of triangles are they ?

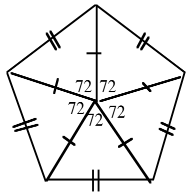
(b) Each triangle has one angle at the centre of the pentagon. The size of this angle is 72˚. Why is it 72˚ ?

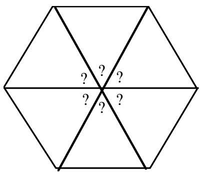
2. <u>Constructing a regular pentagon</u>

Step 1: Draw a circle of radius 6 cm.

Step 2: Construct five 72˚ angles at the centre of the circle. Extend the arms of each angle out to the circle.

Step 3: Join the ends of the arms to make a pentagon.



3. 

This is a regular hexagon.

(a) What is the size of each of the angles at the centre ?

(b) Construct a regular hexagon.

4. A regular nonagon has 9 equal sides and 9 equal angles.
Construct a regular nonagon.

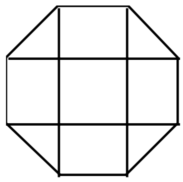
5. Construct a regular decagon (10 sided figure).

6. A regular octagon has 8 sides.

(a) Construct a regular octagon.

(b) Rub out all construction lines, leaving only the octagon.

(c) Draw horizontal and vertical lines in like this.



(d) Are the five quadrilaterals inside this shape squares or rectangles ?

(e) What kind of triangles are the four triangles ?

*7. What regular polygon has 18˚ angles at the centre ?

**8. A regular polygon has 15˚ angles at the centre.

(a) How many sides does it have ?

(b) Work out the size of an interior angle of this polygon.

**9. Another regular polygon has exterior angles of 1˚.
What size angles does it have at the centre ?

• Check your answers.

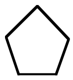
EXTENSION

E12.3: Where do the names come from ?

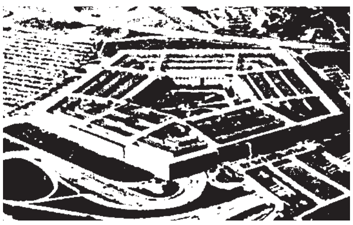
The words used for 2–D and 3–D shapes come from Latin (the language of the Romans) and Ancient Greek (which is not quite the same as modern Greek). Then, just to confuse everyone, some words are a mixture of Latin and Greek.

"Poly" was Greek for "many". A **polygon** is a many sided 2–D shape.
A **polyhedron** is a many sided 3–D shape. The plural is **polyhedra.**

"Penta" is Ancient Greek for five.
A **pentagon** is a five sided polygon.



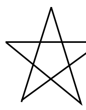
The modern Greek for five is "pembti". However, pentagon and pembtigon sound very alike !



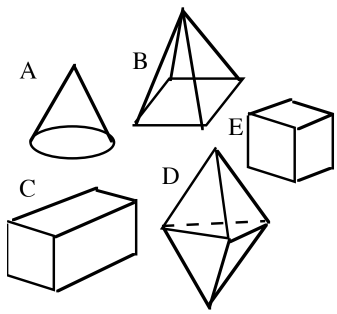
In America, the Pentagon is a large five-sided building which was built in the second World War as the American War Office. When built, it was the largest office building in the world.

It is very clear to spot from the air – and so could be bombed very easily – if any potential enemy was close enough to fly over. In 1941, no planes had the range to reach the Pentagon from any potentially hostile county. At that time, all European War Offices were in underground bunkers or anonymous office buildings.

A pentacle is a 5–pointed star and is often thought to have magical properties.



1. (a) Which of these shapes is a pentahedron (five–sided poyhedron) ?
   (b) What is the more usual name of this pentahedron ?
   (c) Is it possible to have a regular pentahedron ?
   Can you draw it ?



A **regular polyhedron** has faces that are regular polygons of the same shape and size.

The Ancient Greek word for six is " Exi"

A **hexagon** is a 6–sided polygon.

A **hexahedron** is a 6–sided polyhedron.

2. (a) Three of the shapes above are hexahedra. Which three ?
   (b) Which is the regular hexahedron ?
   (c) Why is D *not* a regular hexahedron ?

Did you know that…
… a prawn is a decapod ? It has ten appendages (feet & antennae)

Did you know that…
… a nonagenarian is someone who is between 90 and 100 years old ?

Latin	Number	Greek
unus	1	ena
duo	2	dhia/dyo
tria	3	dria
quattuor	4	tetra
quinque	5	penta
sex	6	exi
septem	7	epta
octa	8	octo
novem/nonus	9	ennea
decem	10	deca
duodecem	12	dhiadeca
	20	eikosi

Did you know that …
… a tetradactylous animal has four digits ?

Did you know that …
…in the medieval monk's day, the service at 12 o'clock was called NONE, because it was at the ninth hour ? The monk's day started with PRIME at 3 a.m. Later this time of day came to be called NOON.

Use the information above to answer the questions below.

How many …

3. … singers or musicians are there in an octet ?
4. … wheels on a quadricycle ?
5. … sides has an icosagon ?
6. … horns has a unicorn ?
7. … carriageways has a dual carriageway ?
8. … corners has a tricorn hat ?
9. … steel plates has an icosphere (a tank for holding volatile liquids)
10. … million in a nonillion ?
11. … strings on a tetrachord ? (a stringed musical instrument)
12. … athletic events are there in a pentathlon ?
13. … feet has a quadruped ?
14. … syllables has a pentasyllabic word ?
15. … islands are there in the Dodecanese ? (Group of islands in the Aegean Sea)
16. A pentarchy is a state or country. with five joint rulers.
    How many rulers does a heptarchy have ?
17. If a road trifurcates it splits into … branches. What is the missing number ?
18. A hexameter is a piece of poetry with six feet (or beats).
    How many feet has a pentameter ?
19. A quadrant is a quarter of a circle. What fraction of a circle is an octant ?
20. A 50p coin is very nearly a heptagon.
    Give one reason why it is not a true heptagon.

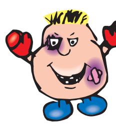
Did you know that…
… 'to unite' means to become one.

Section 13: Angles in polygons

In this section you will calculate angles in polygons.

DEVELOPMENT

D13.1: Exterior angles in a polygon



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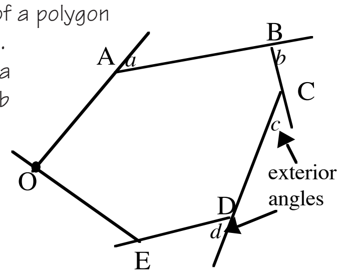
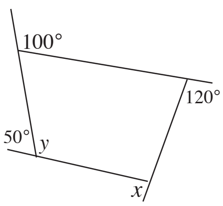
Imagine walking round the sides of a polygon OABCD… until you get back to O.
At A you turn through the angle a
At B you turn through the angle b
At C …

When you get back to O and are facing in the direction OA, you have turned through one complete revolution.



For any polygon the sum of the exterior angles is 360°

Task 1: *Work out the value of each letter.*
Any two angles with the same letter are the same size.

1.


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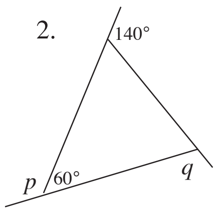

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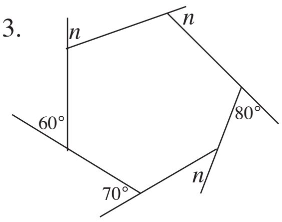

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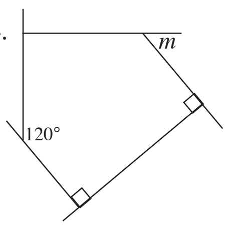

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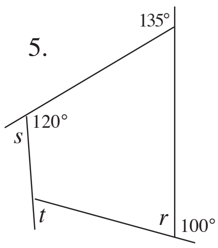

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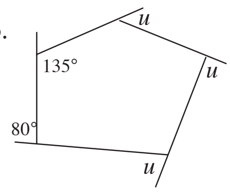
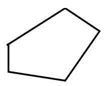


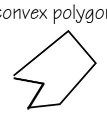
Task 2: The rule should be

The sum of the exterior angles of any convex polygon is 360°

Explain why the rule does not work for concave polygons.



convex polygon



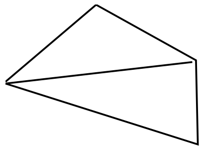
concave polygon

• Check your answers.

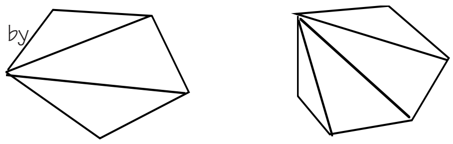
D13.2: The sum of the interior angles of a polygon

Task 1: The sum of the interior angles of any quadrilateral is 360°.

Explain how you can use the diagram to show this.



Polygons can be divided into triangles, by drawing lines from one vertex to every other vertex.



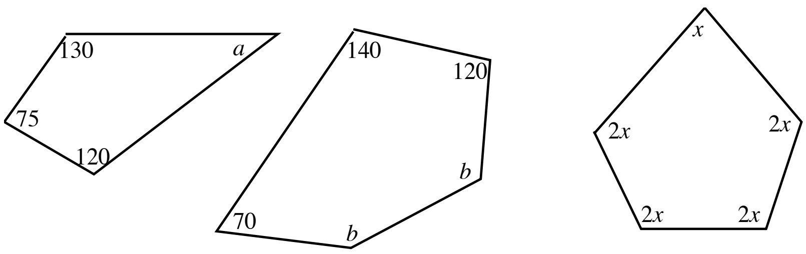
Task 2: *Copy and complete this table:*

No. of sides	No. of triangles	sum of interior angles
4		
5		
6		
7		
8		
20		
50		
n		

Task 3: S = sum of interior angles. Write S as a formula involving n.

• *CHECK YOUR ANSWERS BEFORE GOING ON TO TASK 4.*

Task 4: *Work out the value of each letter:*



Task 5: To be exact, the rule should be written

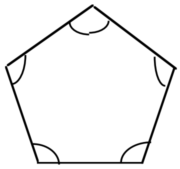
The sum of the interior angles of a convex polygon is $(n - 2) \times 180°$

Explain how you know that the rule does not work for concave polygons.

• *Check your answers.*

Ruff : Topic 6: Coordinates, Shapes and Transformations Section 13 page 35

D13.3: Angles in regular polygons

1. 

<u>Method 1</u> A regular pentagon has 5 equal interior angles and 5 equal exterior angles.

(a) Calculate the size of an exterior angle of a regular pentagon.

(b) Use the answer from (a) to work out the size of an interior angle of a regular pentagon.

2. <u>Method 2</u>

(a) Use the formula (from D13.2) to calculate the sum of the interior angles of a regular pentagon.

(b) Use the answer from (a) to work out the size of an interior angle of a regular pentagon.

Calculate the interior angles of each of these regular polygons.
Work out some using method 1 and some using method 2.

3. regular hexagon
4. regular octagon
5. regular decagon (10 sides)
6. regular dodecagon (12 sides)
7. regular icosagon (20 sides)

8. Work out how many sides has the regular polygon where one exterior angle is 15°
9. Work out how many sides has the regular polygon where one exterior angle is 175°

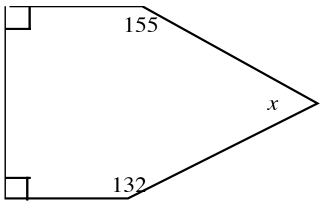
*10. Explain why you cannot have a regular polygon with an interior angle of 138°

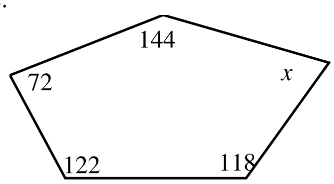
• *Check your answers.*

EXTENSION

E13.3: Polygon angle challenge !

Work out the value of x in each case.
The polygons are not drawn accurately.

1. 

2. 

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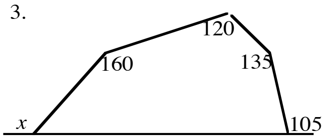

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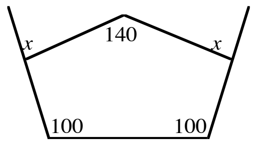

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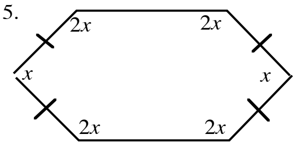

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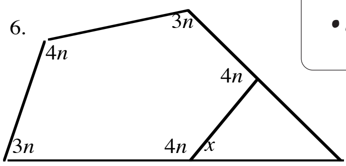
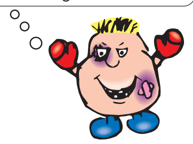


It usually helps to:
- draw a rough sketch of the diagram
- do the working out on the diagram



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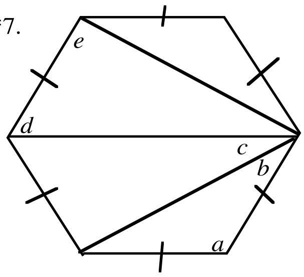
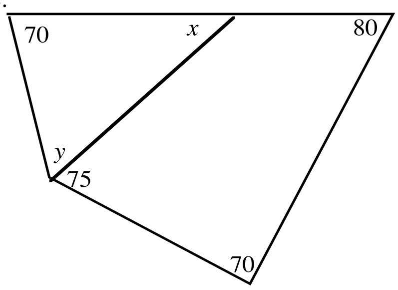
Work out the size of each lettered angle:

*7.



*8.



**9. *Work out the size of:*

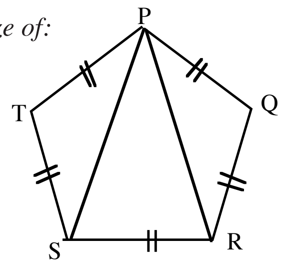
(a) $P\hat{Q}R$

(b) $P\hat{R}Q$

(c) $P\hat{R}S$

(d) $R\hat{S}P$

(e) $R\hat{P}T$



All working must be shown on the diagrams for Q9 and 10

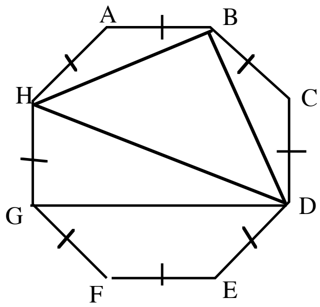
**10. *Work out the size of:*

(a) $H\hat{A}B$ (b) $A\hat{H}B$

(c) $C\hat{B}D$ (d) $H\hat{B}D$

(e) $H\hat{D}B$ (f) $D\hat{G}H$

(g) $G\hat{H}D$



• *Check your answers.*

Section 14: Tessellating polygons

In this section you will:
- calculate whether a given regular polygon will tessellate or not
- find pairs of polygons that will tessellate together

DEVELOPMENT

D14.1: Which regular polygons tessellate ?

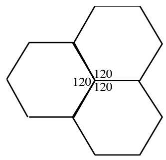
Task 1: *Copy and complete this table*:

Regular polygons										
Number of sides	3	4	5	6	7	8	9	10	11	12
Size of interior angle	60°									

A tessellation is a regular repeating pattern with no gaps.

Regular hexagons tessellate on their own.

Each interior angle is 120°.
120° divides into 360° with no remainder.

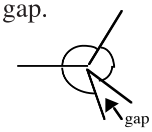


Task 2: Find three other regular polygons that will tessellate on their own. Explain how you know that they will tessellate, without drawing the tessellation.

Task 3: Find two regular polygons that will not tessellate on their own. Calculate the size of the smallest gap in each case.

Task 4: Regular dodecagons (12 sided polygons) will not tessellate on their own. But, there is another regular polygon that will fit in the gap.

(a) Which regular polygon will fit in the gap ?



(b) Explain how you know it will fit in the gap.

(c) Draw a diagram to show how these two polygons tessellate together.

(d) Make a tessellation using at least six of each polygon.

Task 4: (a) Find another pair of regular polygons that will tessellate together. One of the polygons must be a regular octagon.

(b) Explain how you can calculate that they will tessellate.

(c) Make a tessellation using at least six of each polygon.

• Check your answers.

ANSWERS

Unit 6: Coordinates, Shapes …

Section 1: Circles p 2

D1.1: Words associated with circles

1. BE and AD 2. AB and CD 3. XBZ
4. T 5. F 6. T 7. F 8. T
9. T 10. T 11. T 12. T 13. F
14. T 15. T 16. T 17. T 18. F
19. F 20. T 21. F

D1.2: Properties of circles

1. 4 cm 2. 6.6 cm 3. 4 cm 4. 3.5 cm 5. T
6. T 7. F 8. T 9. T 10. T

Section 2: Coordinates and shapes p 4

D2.1: Olly's camp

1. Fox hole 2. badgers 3. (0,5)

4.

Fossils	Cave	Rat skeleton	Badger Sett	Willow
(–1,2)	(4,1)	(0,5)	(4,–1)	(1,4)

Picnic spot	Bog	Foxhole	Stony place	View pt
(1,–3)	(–2,5)	(–5,2)	(–2,–1)	(3,–5)

Old wall	Swamp	Tree house	Lookout point
(–2,–4)	(–4,–3)	(–6,–4)	(–5,–1)

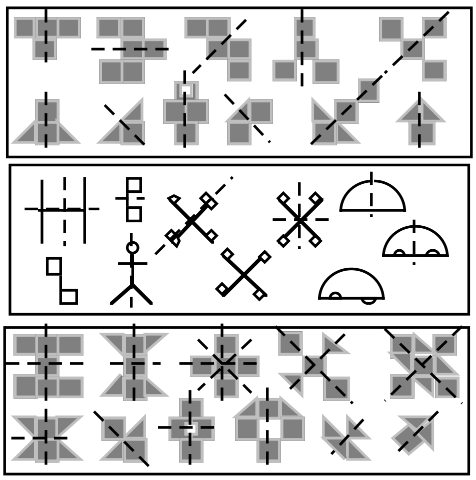
Spring	Camp	Climbing tree	Rock scramble
(–6,5)	(0,0)	(5,–3)	(1,–2)

D2.2: Shapes and coordinates

1. (b) (i) (6,4) (ii) (4,3) 2. (b) (5,8)
3. (b) (i) (1,-5) (ii) (3.5, -4) (iii) (5.5, -4)
4. (b) (i) (-7,5) (ii) (-9.5, 5.5)
5. (i) (10,2) (ii) (9,0) (iii) (10, -0.5)
6. (i) (7,-8,5) (ii) (-6,-1) (iii) (-5,-4)
7. (i) (-7,-6) (ii) (-6,-1) (iii) (-5,-4)
8. (c) (-8, -11)

Section 3: Mirror symmetry p 6

D3.1: Finding lines of symmetry



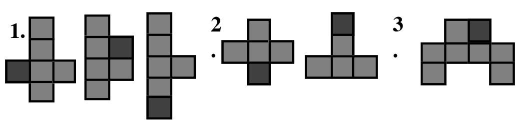
D3.2 : 3-D symmetry

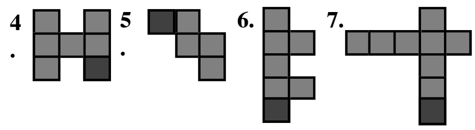
1. 4 2. 3 3. 4 4. 2 5. 1 6. 3
7. 6 8. 2 9. 2

E3.3 : *3-D symmetry

BDFH BCHE ADGF ABGH CDEF

E3.5 : *Add a square





Section 4: Rotational Symmetry p 9

D4.1: Circle Patterns

Task 4: Circle Pattern	A	B	C
Order of rotational symmetry	2	3	1

D4.2: Rotational symmetry

Task 1: shape	R	S	P	C	E	O	T
no. of ways	2	4	2	4	3	8	1

Task 2:	A	B	C	D	E	F	G	H	I	J	K	L
order	1	1	2	2	2	1	4	2	2	1	4	1
lines	1	1	2	0	0	1	0	2	0	1	4	0

D4.3 : Polygon symmetry

letter	S	H	A	I	R	T	E	D	P	K
lines	4	6	1	1	2	0	3	2	0	1
order	4	6	1	1	2	1	3	2	2	1

Section 5: Transformations p 12

D5.1: Which transformation could it be ?

Key: T ≡ translation Re ≡ reflection Ro ≡ rotation E ≡ enlargement

1. Re 2. T 3. E 4. Ro 5. Re 6. T
7. T 8. Re 9. Re 10. T 11. Ro 12. T

Section 6: Reflections p 13

D6.1: Reflections on coordinates

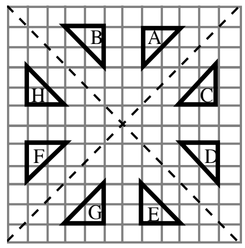
1. C 2. E 3. H 4. F 5. A 6. B 7. B 8. F
9. D 10. F 11. D 12. H 13. B

D6.2: Meeting some simple lines again

1. E 2. B 3. G
4. A is $y = 2$ B is $y = –2$ C is $y = –x$ D is $x = 5$
E is $x = 2$ F is $x = –2$ G is $y = x$ H is $y = 5$

D6.3: Reflective symmetry

ΔA is (1,3) (1,5) (3,5)
ΔB is (-1,3)(-1,5)(-3,5)
ΔC is (3,1)(5,1)(5,3)
ΔD is (3,-1)(5,-1)(5,-3)
ΔE is (1,-3)(1,-5)(3,-5)
ΔF is (-3,-1)(-5,-1)(-5,-3)
ΔG is (-1,-3)(-1,-5)(-3,-5)
ΔH is (-3,1)(-5,1)(-5,3)

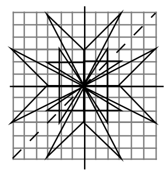


D6.4: Find the mirror lines

A—>B	$y = x$	F—>C	$x = 0$
A—>D	$y = 0$	A—>D	$y = 0$
H—>G	$y = –x$	H—>A	$x = 0$
B—>G	$x = 0$	D—>C	$y = –x$
E—>H	$y = 0$	F—>A	$y = –x$

E6.5: Star reflection challenge

(6,3)(3,0)(2,0)(2,3) (given)
(6,–3)(2,-3)(-2,-3)(-6,-3)
(-6,3)(-2,3)(3,6)(3,-6)
(-3,-6)(3,-2)(-3,-2)(-3,2)



E6.6: Rules for coordinate reflections

2.

A	B	C	D	E	F	G
(3,1)	(9,1)	(11,4)	(8,3)	(6,5)	(4,3)	(1,4)
(3,-1)	(9,-1)	(11,-4)	(8,-3)	(6,-5)	(4,-3)	(1,-4)

3. *x*–coordinate is same.
4. y–coordinate is same number, different sign
7. (5,7) —> (5,–7) (2,–3) —> (2,3) (1,4) —> (1,–4)
(–3,–4) —> (–3,4)

10.

A	B	C	D	E	F	G
(3,1)	(9,1)	(11,4)	(8,3)	(6,5)	(4,3)	(1,4)
(-3,1)	(-9,1)	(-11,4)	(-8,3)	(-6,5)	(-4,3)	(-1,4)

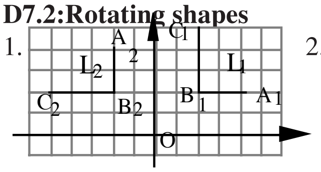
12. (3,2) —> (-3,2) (4,-1) —> (-4,-1) (-2,5) —> (2,5)
(3,4) —> (–3,4) (-3,-2) —> (3,-2)
13. *(x,y)* —> *(–x,y)*

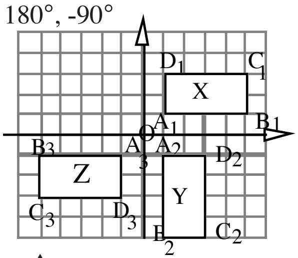
Section 7: Rotations p 19

D7.1: Describing rotations

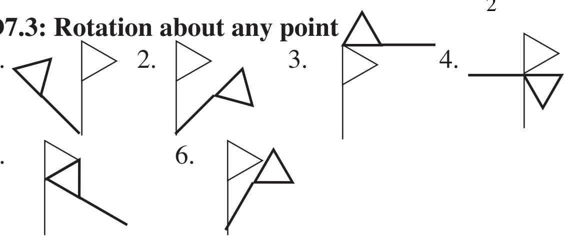
Angles are : 90°, 90°, 180°, 90°, -90°, 180°, -90, 90°
All centres are O.
2. –90°, 180°, -90°, 90°, 180°, 180°, -90°

D7.2:Rotating shapes

1.


2.


D7.3: Rotation about any point



Section 8: Angle properties p 21

D8.1: Angles at a point and on a straight line

1. 120° 2. 160° 3. 50° 4. 50° 5. 140°
6. 80° 7. $g = 70°$, $h = 45°$ 8. $i = 65°$, $j = k = 115°$
9. $m = 110°$ 10. $n = 40°$ 11. $p = 40$ 12. $q = 118°$
13. $r = 30°$ 14. $s = 126°$ 15. $t = 32°$ $u = 148°$
16. $v = 60°$

D8.2: Angles in a triangle

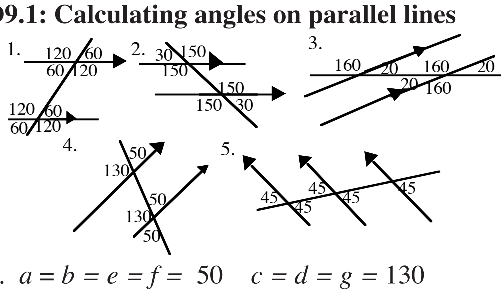
1. 75° 2. 65° 3. 100° 4. 123°
5. 46° 6. 46° 7. 50° 8. 50°

D8.3: Special triangles

1. $a = 60°$ 2. $b = 60°$ 3. $c = 43°$
4. $d = 70°$, $e = 40°$ 5. $f = 20°$ 6. $g = h = 78°$
7. $i = 46°$ 8. $j = 40°$ 9. $k = 55°$, $l = 70°$
10. $m = 30°$, $n = 75°$ 11. $p = q = 70°$, $r = 40°$
12. $t = 45°$

Section 9: Parallel lines p23

D9.1: Calculating angles on parallel lines



6. $a = b = e = f = 50$ $c = d = g = 130$
$u = 60$ $v = 120$

7. $l = m = 40$ $h = 140$ $i = j = k = 65$
8. $q = 60$ $p = r = 120$ 9. $u = v = 25$ $w = 155$
10. $a = 140$ $b = 55$ $c = 95$
11. $d = j = 110$ $e = i = 30$ $f = 70$

D9.2: Classifying angles on parallel lines

1. (a) *m* (b) *b* (c) *g* (d) *c* (e) *b* (f) *i* (g) *h*
2. (a) *p&n* *r&q* (b) *p&s* *q& u* (c) *n&s* *r & u*

P9.3: A mixture of angles

1. $a = b = 40$, $c = 100$
2. $e = h = 20$, $d = g = 80$, $f = 140$
3. $j = 64$, $i = 116$, $k = 145$
4. $p = q = s = 55$ $r = 70$ 5. $m = 117$ $n = 63$ $t = 33$
6. $x = z = 65$, $u = 55$, $v = 70$, $w = 110$, $y = 50$

D9.4: Angles and explanations

1. $a = 65$ $b = 40$ $c = 75$ 2. $d = 110$ $e = 75$
3. $r = 50$ $s = 70$ $t = 120$ 4. $u = 50$
5. $m = 130$ 6. $n = 70$

Section 11: Quadrilaterals p 27

D11.1: Classifying quadrilaterals

1. rhombus 2. trapezium 3. arrowhead
4. kite 5. parallelogram

D11.2: Properties of quadrilaterals

1. T 2. F (one is obviously longer than the other) 3.T
4.F (parallelogram/rectangle also has two pairs of equal sides)
5. T 6. F (one is obviously longer than the other) 7. T
8. T 9. T 10. F (one is obviously longer than the other)
11. T 12. F (no right-angles)
13. F (only 1 pair of parallel sides) 14. T
15. F (O or 2 right angles only) 16. square or rectangle
17. square or rectangle 18. square or rhombus
19. parallelogram 20. trapezium

D11.3: Quadrilateral calculations

1. 40° 50° 2. 50° 20° 3. 75° 105° 80° 25°
4. 66° 24° 132° 5. 55° 35° 55°
6. 70° 30° 80° 20° 7. 50° 115° 65°
8. 40° 110° 110° 9. 32° 74° 64°
10. 65° 130° 65° 85° 11. 60° 30° 15° 75°

E11.4: Completing the decision tree

X is "right angles ?" A = square B = rectangle C = parallelogram D = trapezium
E = kite F = no name G = arrowhead

Section 12: Polygons p 31

D12.1: Properties of some polygons

1. A B D G H 2. P Q V 3. square
4. rhombus 5. equilateral

D12.2: Constructing regular polygons

1. (a) isosceles (b) 360 ÷ 5 = 72 3. (a) 60°
4. centre angle is 40° 5. centre angle is 36°
6. (a) centre angle is 45° (d) 4 rectangles and 1 square
(e) right-angled
7. Icosagon (20 sides) 8. (a) 24 sides (b) 165° 9. 1°

E12.3: Where do the names come from ?

1. (a) B (b) pyramid (c) not possible
2. (a) C, D, E (b) E (c) faces are not regular polygons
3. 8 4. 4 5. 20 6. 1 7. 2
8. 3 9. 20 10. 9 11. 4 12. 5
13. 4 14. 5 15. 12 16. 7 17. 3
18. 5 19. one eighth
20. sides are curved not straight *or* it is not 2-dimensional.

Section 13: Angles in polygons p 35

D13.1: Exterior angles in a polygon

Task 1: 1. $x = 90°$ $y = 130°$ 2. $p = 120°$ $q = 100°$
3. $n = 50°$ 4. $m = 30°$ 5. $s = 60°$ $t = 65°$ 6. $u = 70°$

Task 2: When walking round the sides of a concave polygon you turn through more than 360˚. For any indent, you turn "in" and then "out" through the same angle.

D13.2: The sum of the interior angles of a polygon

Task 1: Sum of interior angles of quadrilateral = sum of interior angles of the two Δs

Task 2:

4	2	360˚
5	3	540˚
6	4	720˚
7	5	900˚
8	6	1080˚
20	18	3240˚
50	48	8640˚
n	$n - 2$	$(n - 2)$x180˚

Task 3: S = $(n - 2)$x180˚

Task 4: $a = 35°$ $b = 105°$ $x = 60°$

Task 5: A concave polygon does not split into Δs in the same way.

D13.3: Angles in regular polygons

1. (a) 72˚ (b) 108˚ (180 – 72)
2. (a) 540˚ (b) 540 ÷ 5 = 108
3. 120˚ 4. 135˚ 5. 144˚ 6. 150˚
7. 162˚ 8. 24 sides 9. 72 sides
10. Ext angle = 42˚ and 42 does not divide into 360.

E13.3: Polygon angle challenge !

1. 73˚ 2. 84˚ 3. 130˚ 4. 80˚ 5. 72˚
6. 60˚ 7. $a = 120°$ $b = 30°$ $c = 30°$ $d = 60°$ $e = 90°$
8. $x = 45°$ $y = 65°$
9. (a) 72˚ (b) 36˚ (c) 72˚ (d) 72˚ (e) 72˚
10. (a) 135˚ (b) 22.5˚ (c) 22.5˚ (d) 90˚ (e) 45˚ (f) 90˚ (g) 67.5˚

Section 14: Tessellating polygons p39

D14.1: Which regular polygons tessellate ?

Task 1:

3	4	5	6	7	8	9	10	11	12
60°	90°	108°	120°	129°	135°	140°	144°	147°	150°

Task 2: triangles, squares, hexagons. Their angles divide exactly into 360°

Task 3: *any two from* : pentagons (gap 36°), heptagons (gap $102\frac{6}{7}$°) octagons (gap 90°) nonagons (gap 80°) decagons (gap 72°) dodecagons (gap 60°)

Task 4: equilateral triangle - the gap is 60°, which is the angle of an equilateral triangle.

Task 5: octagons and squares – the gap is 90°, which is the angle of a square.