



ISBN-10: 1-874428-92-1

ISBN-13: 978-1-874428-92-3



Spoton

# Sample Resources

from the

## Teachers' Resource and Assessment Pack

### for Y11 Spoton (GCSE Higher Tier)

The topic **"Algebra - towards A-Level"**

can be downloaded from the website

[www.mathsisjugglers.com](http://www.mathsisjugglers.com)

**You have permission to print  
this topic for use with your students.**

This pack contains the **Teachers' Resources and Assessments**  
for the topic **"Algebra - towards A-Level"** in the Y11 Spoton Guide.

**You have permission to print these for use with your students.**

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#### **New Higher Syllabus material at D grade level**

*The Spoton texts already cover much of the material at D grade level that is required for the new Higher GCSE Syllabus.*

*However, each school purchasing Spoton texts will be provided with a small package that contains:*

- *a list of all the D grade items*
- *a match of most of these items to the Spoton texts*
- *photocopiable material from our Intermediate texts for the few items that are not covered, for teachers to use as they wish.*

**But -any school can download this package  
free from our website .**

# Contents

## of the

# Higher GCSE Course

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ISBN-10: 1-874428-91-3

ISBN-13: 978-1-874428-91-6

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ISBN-13: 978-1-874428-92-3

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The emphasis here is on non-calculator skills, with a particular stress on mental agility. The Teachers' Guide for each topic (in the Resource Pack) includes a list of items that could be used as mental/oral starters and the techniques taught/reviewed here should be repeated regularly over the weeks following their introduction. For details of Teachers' Resource Packs, see the next page.

The course should start with **Topic 1**. This contains the number techniques that will be assumed thereafter throughout the course.

**The rest of the topics are independent and can be done in any order.**

Any techniques required within a topic that are taught elsewhere, will be repeated at the point where they are required.



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Order one **half price** copy of **Y10 Spoton** and **Y11 Spoton**  
 using the **Special Offer form**  
 which can be **downloaded**  
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# Cross-Topic Information

The emphasis here is on non-calculator skills, with a particular stress on mental agility. Where applicable, sections within the topics open with items that can be used as mental/oral starters and the techniques taught/reviewed here should be repeated regularly over the weeks following their introduction.

Each lesson should start with a brief mental/oral starter. As well as developing mental arithmetic expertise, it provides a positive start to the lesson.

Suggestions for these are given on the topic pages that follow.

The aim is to develop a learning ethos where:

- mental techniques are a first resort
- pencil-and paper techniques are used routinely
- standard arithmetic techniques are used on a regular basis
- non-standard arithmetic techniques (jottings) are acceptable provided they are clearly shown
- calculators are used only when the calculations become complex
- calculator functions are understood and used effectively

Calculators to be used should have, as a minimum, the following functions:

+   -   x   ÷    $x^2$     $\sqrt{x}$    memory   brackets  
 $x^y$  or  $y^x$     $x^{1/y}$  or  $\sqrt[x]{y}$    sin   cos   tan

Students who will take the Intermediate Tier at GCSE will have a wide spread of abilities. It is not expected that every student should do every exercise.

The better students will not need to do the easier exercises. For these students, a brief resumé of a technique, done as an oral starter, is all that is required.

The less able students are not expected to do the extension exercises – but they should be allowed to tackle them, if they feel able to do so.

**The course should start with Topic 1.** This topic contains the number techniques that will be assumed thereafter throughout the course.

**The rest of the topics are independent and can be done in any order.** Any techniques required within a topic that are taught elsewhere will be repeated at the point where they are required.



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The Y10 and Y11 Spoton Guides deliver both the National Curriculum and the linear GCSE syllabuses for AQA, Edexcel and OCR.

However, the topics have been ordered in such a way as to make the material accessible to teachers and students following one of the modular GCSE syllabuses. To facilitate this, two of the topics are in both the Y10 text and the Y11 text. The topic contents of the whole course is listed on the contents page of both texts.

# Topic 11: Algebra – towards A-Level

Printing List

None

Page in text

–

Pack page

–

**In this topic students will review and extend algebraic techniques met in previous years, including “proof”.**

*By the end of the topic, each student should have an excellent grasp of more complex algebra techniques and be well prepared for A-Level.*

## **Suggested mental/oral topics**

Squares of integers between -15 and 15.	
Cubes of integers between -5 and 5, also $\pm 10$	
Substituting numbers into algebraic expressions	(D1.2)
Rearrangements of simple formulae	(D3.1)
Solving simple inequations	(D6.3)
Solving simple orderings of the form $a < f(x) < b$	(D6.6)
Expanding brackets	(D8.1, D8.2)
Expanding $(ax + b)^2$	
Factorising the difference of two squares	(D11.1)
Factorising simple expressions	(D12.1, D12.2)
Evaluating powers	(D15.1)
Simplifying powers	(D15.2)

*As well as*

The mental arithmetic techniques met earlier in the course

By now, the student should be doing mental calculations needing several stages and combining several techniques.

## **Miscellaneous word and number problems**

There are a selection of these at the back of this Teachers' Guide (after Topic 15)

This selection includes those provided in the Y10 R&A Pack, plus many more.

## **Direct teaching points**

All teaching points are in the text.

## **When students are referred to their teacher for answers**

There are no teacher answers for this topic.

## Miscellaneous word problems for use in mental/oral sessions

### Exercise 13

- $15^2 - 3^3 + 7$
- What is the largest multiple of 4 less than 547 ?
- The perimeter of a square is 56 cm. What is the area of the square ?
- $x = 2y$  and  $y = 10$ .  
What is the value of  $x^2 + y^2$  ?
- You are told that  $32 \times 44 = 1408$ . What is the value of  $32 \times 34$  ?
- $7385 - 850 + 7^2$
- In a game of darts, I need to score 501 to win. I have scored 453. I have two darts left. With one I score 16. What double do I need to get to finish ?
- Half of 3.05 + double 2.92
- Solve  $2x - 13 = 47$
- In Scrabble, when I added on my last score, my total went from 269 to 303. My last score was a double word score. What was the score for the word before it was doubled ?

### Exercise 14

- A rectangle is twice as long as it is wide. Its area is  $32 \text{ cm}^2$ . What is the width of the rectangle ?
- $x + y = 7$  and  $x - y = 1$ . What are the values of  $x$  and  $y$  ?
- $1946 \div 7$
- I spent 75% of my savings. I had £12 left. How much did I have saved ?
- Add the multiple of 5 before 679 to the odd number after 789.
- Solve  $3(x - 5) = 39$
- In Scrabble, when I added on my last score, my total went from 129 to 186. My last score was a treble word score. What was the score for the word before it was doubled ?
- The area of a square is  $6.25 \text{ cm}^2$ . What is the length of the side of the square ?
- 13 more than  $2897 + 13^2$ .
- The cost of three rides are : Big Wheel £1.80, Laser £2.60 and Bomber £2.30.  
Phil went on each one twice. How much did that cost ?

### Exercise 15

- $4285 \times 5$
- A rectangle is twice as long as it is wide. Its perimeter is 48 cm. What is the width of the rectangle ?
- $(12^2 - 11^2)^2$
- 480 marbles are put into 8 bags. How many marbles are there in three of these bags ?
- $\frac{4}{5}$  of a number is 84. What is the number ?
- A football team played 15 games. They won 60% of them . How many did they lose ?
- Half of £2.48 + double £1.20.
- I started a book yesterday. Today I read ten more pages than yesterday. I have reached page 70. How many pages did I read yesterday?
- 347 more than  $578 + 7^2$
- $\frac{1}{3}$  of  $\frac{2}{5}$  of £60

### Exercise 16

- I think of a number and subtract 15.  
I square the answer I get and add 7. My final answer is 43.  
What number did I think of ?
- The price of a CD goes up from £10 to £12.  
What is the percentage increase ?
- Work out 17.5% of £80.
- $\frac{5}{6}$  of a number is 25. What is the number ?
- There are 5 stacks of plates. 3 stacks have 25 plates in each stack. 2 stacks have 18 plates in each stack. How many plates are there ?
- Divide 285 by 5 and add 17.
- Angles at a point add up to  $360^\circ$ . Four angles meet at a point. Three of the angles are  $75^\circ$ ,  $120^\circ$  and  $123^\circ$ . What is the fourth angle ?
- Solve  $2(x - 3) + 5 = 13$
- The area of a triangle is given by  $A = \frac{1}{2}bh$ .  
The area of a particular triangle is  $15 \text{ cm}^2$ .  
Its base is 5 cm long. What is its height ?
- Add the HCF of 20 and 36 and the difference between  $6^2$  and  $5^2$ .

# Algebra – towards A-Level REVISION

Section 15

12. Write as powers of  $x$  or sums of powers of  $x$ :
- (a)  $(x^{1/2})^3$  (b)  $x^2 + x\sqrt{x}$  (c)  $x\sqrt{x^3}$

Name : .....

Do the answers to this revision sheet in your exercise book or on paper – NOT ON THIS SHEET. Check your answers using the answers on the reverse of this sheet. KEEP THIS SHEET SOMEWHERE SAFE. USE IT AGAIN TO REVISE FOR EXAMS.

**In tests/exams, all working must be clearly shown.**

Section 1

1. Find the value of each expression when  $n = -2$  and  $m = 3$ :

(a)  $n^2 + m$  (b)  $2m^3 + 5n$  (c)  $(m - 2n)^2$  (d)  $3n^3 + 2m^2$

Section 2

2. Solve each equation. Give answers as whole numbers or fractions.

(a)  $2(a - 3) = 6 - 3a$  (b)  $\frac{n}{2} + 3 = 5$  (c)  $\frac{6a - 7}{2} = 8$

(d)  $\frac{5a}{8} = 7$  (e)  $\frac{u+1}{u-2} = \frac{7}{6}$  (f)  $\sqrt{(x-3)} = 5$

Section 3

3. Make the letter  $n$  the subject of the formula:

(a)  $s = 3n + t$  (b)  $\frac{n - 2a}{3} = 5$  (c)  $\frac{7}{n} = 3t$  (d)  $\frac{a + 3n}{b} = t$

Section 4

4. Make the letter  $x$  the subject of the formula:

(a)  $ax - b = c - dx$  (b)  $3(x - a) = px$

Section 5

5. If  $A = \pi r^2$  and  $r + x = 15$ , find  $A$  in terms of  $x$ .

Section 6

6. Solve each inequality:

(a)  $4 - 3x > -5$  (b)  $-2 < 2x - 1 < 3$  (c)  $x < 2x + 7 < 15$

Section 7

7. Solve using the elimination method:

(a)  $3x + 2y = 13$  (b)  $4x + y = 6$  (c)  $2x + 3y = 18$   
 $5x - 2y = -5$  (b)  $9x - 4y = 1$  (c)  $9x - 4y = 11$

Section 7

8. Solve using the substitution method:

(a)  $3x + 4y = 23$  (b)  $4x - 2y = 2$  (c)  $2x + 3y = 13$   
 $y = 2x + 3$  (b)  $y = x + 1$  (c)  $x + y = 6$

Section 9

9. Solve using any method you choose:

(a)  $6x + 5y = 35$  (b)  $7x - 4y = 1$  (c)  $x + 2y = 10$   
 $x - 2y = 3$  (b)  $3x - y = 9$  (c)  $3x - y = 9$

Section 10

10. Solve using the substitution method:

$3y^2 - x^2 = y - 2$ ,  $x = y + 1$

Section 8

11. Multiply out all brackets and simplify:

(a)  $2(x + 3)^2$  (b)  $(2x + 3)^2$  (c)  $(x - 4)^2 - (x + 6)^2$  (d)  $\left[ x + \frac{1}{x} \right]^2$

For each of the following sets of techniques, a lot of practice has been provided. YOU decide how much of each kind of practice you need to do.

Sections 9,11,12

**Technique set 1 – Factorise:**

13.  $x^2 + 2x$  14.  $x^2yz^2 - xyz^3$  15.  $x^4 + x^3 + x^2$   
 16.  $x^2 + 5x + 4$  17.  $x^2 - 10x + 16$  18.  $x^2 + 3x - 18$   
 19.  $5x^2 - 6x + 1$  20.  $8x^2 - 2x - 1$  21.  $7x^2 - 3x - 4$   
 22.  $x^2 - y^2$  23.  $x^2 - 4y^2$  24.  $9x^2 - 25$   
 25.  $4p^2 - q^2r^2$  26.  $1 - 9m^2$  27.  $t^4 - m^4$  (3 factors)

Section 9

**Technique set 2 – Solve, using factorisation;**

28.  $x^2 + 8x + 15 = 0$  29.  $x^2 - 10x + 21 = 0$  30.  $x^2 + x - 12 = 0$   
 31.  $2x^2 + 3x - 2 = 0$  32.  $9x^2 - 3x - 2 = 0$  33.  $2x^2 + 3x - 35 = 0$

Section 13

**Technique set 3 – Solve, by completing the square:**

34.  $x^2 + 6x - 27 = 0$  35.  $x^2 + 6x + 5 = 0$  36.  $x^2 - 4x + 1 = 0$   
 37.  $x^2 - 10x + 4 = 0$  38.  $x^2 + x - 3 = 0$  39.  $x^2 - 3x - 2 = 0$

Section 14

**Technique set 4 – Solve, using the formula:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$**   
 (Give exact answers or to 2 d.p.)

40.  $x^2 - 4x + 3 = 0$  41.  $x^2 + 8x - 20 = 0$  42.  $2x^2 - 7x + 3 = 0$   
 43.  $4x^2 + 11x - 3 = 0$  44.  $3x^2 - 2x - 6 = 0$  45.  $5x^2 - 9x + 1 = 0$

Section 14

**Technique set 5 – Solve, using any method you like:**

46.  $2x^2 - 3x + 1 = 0$  47.  $6x^2 + 1 = 5x$  48.  $4x + \frac{3}{x} = 7$   
 49.  $\frac{5x+3}{2} = x^2$  50.  $x + 5 = \frac{14}{x}$  51.  $x^2 + 3x = 0$

Section 3

**Technique set 6 – Make  $v$  the subject of each formula:**

52.  $Cv = m + p$  53.  $pv + t = m$  54.  $av^2 - b = c$   
 55.  $\frac{v}{2} + m = 3n$  56.  $2(u + v) = 3t$  57.  $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

**Technique set 7 – Factorise and simplify:**

58.  $\frac{2x^2 - x}{2x}$     59.  $\frac{5 - 15x^2}{5x}$     60.  $\frac{4 + 8x + 12x^2}{4x}$   
 61.  $\frac{x^2 + 3x}{x^2 - 2x}$     62.  $\frac{x^2 - x - 2}{x^2 + x - 6}$     63.  $\frac{x^2 + 3x}{x^2 + 2x - 3}$

**Technique set 8 – Write as a single fraction:**

64.  $\frac{x+3x}{5}$     65.  $\frac{5}{x} + \frac{1}{x}$     66.  $\frac{2}{3x} + \frac{1}{6x}$   
 67.  $\frac{x+1}{2} + \frac{x}{3}$     68.  $\frac{x-1}{2} + \frac{x+1}{4}$     69.  $\frac{1}{x} + \frac{1}{x+1}$   
 70.  $\frac{x}{x+1} + \frac{3x}{x-1}$     71.  $\frac{3}{x-2} - \frac{2x}{x-1}$     72.  $\frac{x+1}{x+2} - \frac{x}{x-1}$

**Technique set 9 – Factorise and simplify:**

73.  $\frac{x-1}{3} \times \frac{6}{x^2-1}$     74.  $\frac{3x-2}{x} \times \frac{x^2}{x-2}$     75.  $\frac{x^2+x-12}{x^2-3x}$

Section 12

1. (a) 7    (b) 44    (c) 49    (d) -6  
 2. (a)  $12/5$     (b) 10    (c)  $23/6$     (d)  $56/5$     (e) 20    (f) 28  
 3. (a)  $n = (s-t)/3$     (b)  $n = 15 + 2a$     (c)  $n = 7/3t$     (d)  $n = (bt-a)/3$   
 4. (a)  $x = \frac{b+c}{a+d}$     (b)  $x = \frac{3a}{3-p}$   
 5.  $A = \pi(15-x)^2$   
 6. (a)  $x < 3$     (b)  $-1/2 < x < 2$     (c)  $-7 < x < 4$   
 7. (a) 1, 5    (b) 1, 2    (c) 3, 4  
 8. (a) 1, 5    (b) 2, 3    (c) 5, 1  
 9. (a) 5, 1    (b) 7, 12    (c) 4, 3  
 10.  $x = 1.5, y = 0.5$  &  $x = 2, y = 1$   
 11. (a)  $2x^2 + 12x + 18$     (b)  $4x^2 + 12x + 9$     (c)  $-20x - 20$     (d)  $x^2 + 2 + 1/x^2$   
 12. (a)  $x^{3/2}$     (b)  $x^{-1} + x^{-3/2}$     (c)  $x^{4/3}$

Section 17

13.  $x(x+2)$     14.  $xyz^2(x-z)$     15.  $x^2(x^2+x+1)$     16.  $(x+4)(x+1)$   
 17.  $(x-2)(x-8)$     18.  $(x+6)(x-3)$     19.  $(5x-1)(x-1)$     20.  $(4x+1)(2x-1)$   
 21.  $(7x+4)(x-1)$     22.  $(x-y)(x+y)$     23.  $(x-2y)(x+2y)$     24.  $(3x+5)(3x-5)$   
 25.  $(2p-qr)(2p+qr)$     26.  $(1-3m)(1+3m)$     27.  $(l+m)(l-m)(l^2+m^2)$   
 28. -3, -5    29. 3, 7    30. 3, -4    31. -2,  $1/2$     32.  $2/3, -1/3$     33. -5,  $3/2$   
 34. 2.2, -8.2    35. -1, -5    36. 3.73, 0.27    37. 0.42, 9.58    38. 1.3, -2.3  
 39. 3.56, -0.56  
 40. 1, 3    41. -10, 2    42.  $1/2, 3$     43. -3,  $1/4$     44. 1.78, -1.12    45. 1.68, 1.19  
 46. 1.68, 0.12    47.  $1/2, 1/3$     48.  $1, 3/4$     49.  $-1/2, 3$     50. 2, -7    51. 0, -3  
 52.  $v = \frac{m+p}{C}$     53.  $v = \frac{m-l}{p}$     54.  $v = \sqrt{\frac{b+c}{a}}$     55.  $v = 2(3n-m)$   
 56.  $v = \frac{3t-u}{2}$  or  $v = \frac{3t-2u}{2}$     57.  $v = \frac{uf}{u-f}$   
 58.  $\frac{2x-1}{2}$     59.  $\frac{1-3x^2}{x}$     60.  $\frac{1+2x+6x^2}{x}$     61.  $\frac{x+3}{x-2}$     62.  $\frac{x+1}{x+3}$     63.  $\frac{x}{x-1}$   
 64.  $\frac{4x}{5}$     65.  $\frac{6}{x}$     66.  $\frac{5}{6x}$     67.  $\frac{5x+3}{6}$     68.  $\frac{3x-1}{4}$     69.  $\frac{2x+1}{x(x+1)}$   
 70.  $\frac{4x^2+2x}{x^2-1}$     71.  $\frac{7x-2x^2-3}{(x-2)(x+1)}$     72.  $\frac{-1-2x}{(x-2)(x+1)}$   
 73.  $\frac{2}{x+1}$     74.  $\frac{3x}{x+1}$     75.  $\frac{x+4}{x}$

# Algebra – Towards A-Level ASSESSMENT

Overall mark = %

Name : .....

Write the answers on this sheet. All working must be clearly shown.

1. Solve by elimination:

$$2x + y = 5$$

$$3x - 2y = 4$$

(6 marks)

2. Solve by elimination:

$$7d + 5e = 32$$

$$3d + 4e = 23$$

(6 marks)

3. Solve by substitution:

$$2m = 4 + p$$

$$6m - 4p = 18$$

(6 marks)

4. Solve each equation:

(a)  $4(x - 1) = 2(3 - x)$

(b)  $\frac{4a + 6}{5} = 10$

(3 marks for each : total 12)

(c)  $5 = \frac{3}{x}$

(d)  $\frac{x-3}{x+1} = \frac{7}{9}$

5. Solve each inequality: (3 marks for each : total 9)

(a)  $4 - 2x < 2$

(b)  $-3 \leq x + 5 \leq 3$

(c)  $x < 3x - 1 < 2x + 7$

6. Make the letter  $a$  the subject of each formula: (3 marks for each : total 9)

(a)  $v = u + at$

(b)  $\frac{a}{a} = k$

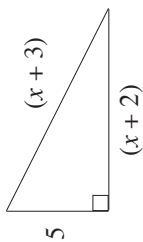
(c)  $\frac{da + e}{c} = f$

7. Make the letter  $x$  the subject of the formula: (4 marks)

$$3x + 2y = y(5 - x)$$

8. If  $S = 2\pi r^2 h + 2\pi r^2$ , find  $h$  in terms of  $S$  and  $r$ . (4 marks)

9. Use Pythagoras' Theorem to make an equation in  $x$ . Solve for  $x$ . (4 marks)



10. Write in the form  $x^n$  or  $ax^n + bx^m$ :

(a)  $(x^3 \times x^4)^2 = \dots\dots\dots$  (2 marks)

(b)  $\frac{x^3}{\sqrt{x}} = \dots\dots\dots$  (2 marks)

(c)  $\frac{6\sqrt{x+4x^3}}{2\sqrt{x}} = \dots\dots\dots$  (4 marks)

11. Factorise:

(2 marks for each : total 8)

(a)  $m^2 - 4m - 21 = \dots\dots\dots$

(b)  $3b^2 - 17b - 28 = \dots\dots\dots$

(c)  $9x^2 - 1 = \dots\dots\dots$

(d)  $4c^2 - 2cd = \dots\dots\dots$

12. Factorise and simplify:

(4 marks for each : total 16)

(a)  $\frac{4x^2 - 2x}{2x} = \dots\dots\dots$

(b)  $\frac{2x^2 - x - 3}{4x^2 - 4x - 3} = \dots\dots\dots$

(c)  $\frac{2x^2 - 32}{2x^2 + 9x + 4} = \dots\dots\dots$

(d)  $\frac{x^3 - 5x^2}{x^3 - 4x^2 - 5x} = \dots\dots\dots$

13. Solve:  $x^2 + y^2 = xy + 19$  and  $y = x + 2$  (6 marks)

14. Solve by factorisation: (3 marks)

$8x^2 - 10x + 3 = 0$

15. Solve, using the formula: (3 marks)

$2x^2 + 5x - 1 = 0$  (Ans to 2 d.p.)

16. Solve by completing the square: (3 marks)

$x^2 - 6x - 11 = 0$  (Ans to 2 d.p.)

17. Solve using any method: (3 marks)

$2x^2 + 3x - 2 = 0$  (Ans to 2 d.p.)

18. Solve  $4x + \frac{7}{x} = 29$  (4 marks)

19. Simplify:

(2 marks each: total 8)

(a)  $\frac{9 + 18x}{6x} = \dots\dots\dots$  (b)  $\frac{1}{7x} + \frac{2}{3x} = \dots\dots\dots$

(c)  $\frac{3}{x} + \frac{2}{x-1} = \dots\dots\dots$

(d)  $\frac{w^3y^2}{w} + \frac{wv}{w^2} = \dots\dots\dots$

OVERALL MARK = / 122

# Algebra – Towards A-Level

## ASSESSMENT ANSWERS

Overall mark = %

Write the answers on this sheet. All working must be clearly shown.

1. Solve by elimination:

$$\begin{aligned} 2x + y &= 5 \\ 3x - 2y &= 4 \\ 4x + 2y &= 10 \\ 3x - 2y &= 4 \\ 7x &= 14 \end{aligned}$$

$$x = 2 \quad y = 1$$

(6 marks)

2. Solve by elimination:

$$\begin{aligned} 7d + 5e &= 32 \\ 3d + 4e &= 23 \\ 21d + 15e &= 96 \\ 21d + 28e &= 161 \\ 13e &= 65 \end{aligned}$$

$$e = 5 \quad d = 1$$

(6 marks)

3. Solve by substitution:

$$\begin{aligned} 2m &= 4 + p \\ 6m - 4p &= 18 \\ 3(4 + p) - 4p &= 18 \\ 12 + 3p - 4p &= 18 \\ 12 - p &= 18 \end{aligned}$$

$$p = -6 \quad m = -1$$

(6 marks)

4. Solve each equation:

(a)  $4(x - 1) = 2(3 - x)$

$$\begin{aligned} 4x - 4 &= 6 - 2x \\ 6x &= 10 \end{aligned}$$

$$x = \frac{5}{3}$$

(b)  $\frac{4a + 6}{5} = 10$

$$\begin{aligned} 4a + 6 &= 50 \\ 4a &= 44 \end{aligned}$$

$$a = 11$$

(3 marks for each : total 12)

(c)  $5 = \frac{2}{x}$

$$5x = 2$$

$$x = \frac{2}{5}$$

(d)  $\frac{x-3}{x+1} = \frac{7}{9}$

$$\begin{aligned} 9(x-3) &= 7(x+1) \\ 9x - 27 &= 7x + 7 \\ 2x &= 34 \end{aligned}$$

$$x = 17$$

(3 marks for each : total 9)

(c)  $x < 3x - 1 < 2x + 7$

$$\begin{aligned} x < 3x - 1 &\Rightarrow 2x > 1 \\ \Rightarrow x > \frac{1}{2} \\ \text{and } 3x - 1 < 2x + 7 \\ \Rightarrow x < 8 \end{aligned}$$

$$\frac{1}{2} < x < 8$$

(3 marks for each : total 9)

(c)  $\frac{da + e}{c} = f$

$$da + e = cf$$

$$da = cf - e$$

$$a = \frac{cf - e}{d}$$

5. Solve each inequality:

(a)  $4 - 2x < 2$

$$\begin{aligned} 4 < 2x + 2 \\ 2 < 2x \end{aligned}$$

$$x < 1$$

(b)  $-3 \leq x + 5 \leq 3$

$$-8 \leq x \leq -2$$

6. Make the letter  $a$  the subject of each formula:

(a)  $v = u + at$

$$at = v - u$$

$$a = \frac{v - u}{t}$$

(b)  $\frac{n}{a} = k$

$$n = ak$$

$$a = \frac{n}{k}$$

7. Make the letter  $x$  the subject of the formula:

$$3x + 2y = y(5 - x)$$

$$3x + 2y = 5y - yx$$

$$3x + yx = 5y$$

$$x(3 + y) = 5y$$

$$\Rightarrow x = \frac{5y}{3 + y}$$

(4 marks)

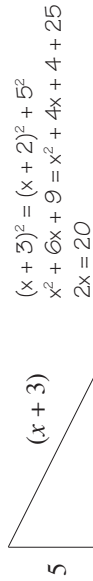
8. If  $S = 2\pi r h + 2\pi r^2$ , find  $h$  in terms of  $S$  and  $r$ .

$$2\pi r h = S - 2\pi r^2$$

$$h = \frac{S - 2\pi r^2}{2\pi r} \quad \text{or} \quad \frac{S - r}{2\pi r}$$

(4 marks)

9. Use Pythagoras' Theorem to make an equation in  $x$ . Solve for  $x$ . (4 marks)



$$x = 10$$

10. Write in the form  $x^n$  or  $ax^n + bx^m$ :
- (a)  $(x^3 \times x^4)^2 = \dots\dots\dots (x^{-1})^2 = \boxed{x^{-2}}$  (2 marks)
- (b)  $x^3 \div \sqrt{x} = \dots\dots\dots x^3 + x^{1/2} = \boxed{x^{5/2}}$  (2 marks)
- (c)  $6\sqrt{x+4x^3} \div 2x = \dots\dots\dots \boxed{3x^{-1} + 2x^{-9/2}}$  (4 marks)

11. Factorise: (2 marks for each : total 8)
- (a)  $m^2 - 4m - 21 = \dots\dots\dots \boxed{(m+3)(m-7)}$
- (b)  $3b^2 - 17b - 28 = \dots\dots\dots \boxed{(3b+4)(b-7)}$
- (c)  $9x^2 - 1 = \dots\dots\dots \boxed{(3x-1)(3x+1)}$
- (d)  $4c^2 - 2cd = \dots\dots\dots \boxed{2c(2c-d)}$

12. Factorise and simplify: (4 marks for each : total 16)
- (a)  $\frac{4x^2 - 2x}{2x} = \dots\dots\dots \boxed{2x - 1}$
- (b)  $\frac{2x^2 - x - 3}{4x^2 - 4x - 3} = \frac{\cancel{2x-3}(x+1)}{(2x+1)\cancel{(2x-3)}} = \frac{\boxed{x+1}}{\boxed{2x+1}}$
- (c)  $\frac{2x^2 - 32}{2x^2 + 9x + 4} = \frac{\cancel{2(x-4)}(x+4)}{(2x+1)\cancel{(x+4)}} = \frac{\boxed{2(x-4)}}{\boxed{2x+1}}$
- (d)  $\frac{x^3 - 5x^2}{x^3 - 4x^2 - 5x} = \frac{x^2(x-5)}{x(x+1)\cancel{(x-5)}} = \frac{\boxed{x}}{\boxed{x+1}}$

13. Solve:  $x^2 + y^2 = xy + 19$  and  $y = x + 2$  (6 marks)
- $x^2 + (x+2)^2 = x(x+2) + 19$   
 $x^2 + x^2 + 4x + 4 = x^2 + 2x + 19$   
 $x^2 + 2x - 15 = 0$   
 $(x-3)(x+5) = 0$  =  $\boxed{x = 3 \text{ or } -5}$

14. Solve by factorisation: (3 marks)
- $8x^2 - 10x + 3 = 0$   
 $(2x-1)(4x-3) = 0$   
 $x = \boxed{1/2 \text{ and } 3/4}$

15. Solve, using the formula: (3 marks)
- $2x^2 + 5x - 1 = 0$  (Ans to 2 d.p.)  
 $x = \frac{-5 \pm \sqrt{(25 + 4 \times 2 \times 1)}}{4}$   
 $x = \frac{-5 \pm \sqrt{33}}{4}$   
 $x = \boxed{0.19 \text{ \& } -2.69}$

16. Solve by completing the square: (3 marks)
- $x^2 - 6x - 11 = 0$  (Ans to 2 d.p.)  
 $[(x-3)^2 - 9] - 11 = 0$   
 $(x-3)^2 = 20$   
 $x-3 = \pm\sqrt{20}$   
 $x = \boxed{3 \pm \sqrt{20}}$   
 $x = \boxed{7.47 \text{ \& } -1.47}$

17. Solve using any method: (3 marks)
- $2x^2 + 3x - 2 = 0$  (Ans to 2 d.p.)  
 $(2x+1)(x-2) = 0$   
 $x = \boxed{-1/2 \text{ \& } 2}$

18. Solve  $4x + \frac{7}{x} = 29$  (4 marks)
- $4x^2 + 7 = 29x$   
 $4x^2 - 29x + 7 = 0$   
 $(4x-1)(x-7) = 0$   
 $x = \boxed{1/4 \text{ \& } 7}$

19. Simplify: (2 marks each: total 8)
- (a)  $\frac{9 + 18x}{6x} = \dots\dots\dots \boxed{\frac{3+6x}{2x}}$
- (b)  $\frac{1}{7x} + \frac{2}{3x} = \frac{3}{21x} + \frac{14}{21x} = \frac{\boxed{17}}{\boxed{21x}}$
- (c)  $\frac{3}{x} + \frac{2}{x-1} = \frac{3(x-1) + 2x}{x(x-1)} = \frac{\boxed{5x-3}}{\boxed{x(x-1)}}$
- (d)  $\frac{u^3v^2}{w} \div \frac{uv}{w^2} = \frac{uv^2}{w} \times \frac{w^2}{uv} = \frac{\boxed{u^2vw}}{\boxed{w}}$

OVERALL MARK = / 122