

The 'Maths is ...' Jugglers

Knowledge

Skills

Understanding



ISBN-10: 1-874428-91-3
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Spoton

Sample Resources

from the

Teachers' Resource and Assessment Pack

for Y10 Spoton (GCSE Higher Tier)

The topic **"Analysing Statistics"**
can be downloaded from the website
www.mathsisjugglers.com

**You have permission to print
this topic for use with your students.**

This pack contains the **Teachers' Resources and Assessments**
for the topic **"Analysing Statistics"** in the Y10 Spoton Guide.
You have permission to print these for use with your students.

CONTENTS OF THE PACK

The structure of the course	p2-3
Teachers' Notes (including teachers answers where not given in the text)	p4
One page from 'Miscellaneous word problems for use in mental/oral sessions'	p5
Revision Sheet (<i>The student can check this using the answers on the back.</i>)	p6-7
Assessment (<i>with spaces for answers on the sheet</i>)	p8-9
Assessment Answers and Mark Schemes (<i>Answers are given on a copy of the test itself – this makes marking much quicker</i>)	p10-11

New Higher Syllabus material at D grade level

The Spoton texts already cover much of the material at D grade level that is required for the new Higher GCSE Syllabus.

However, each school purchasing Spoton texts will be provided with a small package that contains:

- *a list of all the D grade items*
- *a match of most of these items to the Spoton texts*
- *photocopiable material from our Intermediate texts for the few items that are not covered, for teachers to use as they wish.*

**But - any school can download this package
free from our website .**

Contents

of the

Higher GCSE Course



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THE SPOTON GUIDE PART ONE (Y10)

Topic 1: Number Techniques	<i>Covers the techniques that will be assumed in the other topics</i>	5	–	37
Topic 2: Sequences and Proof		38	–	74
Topic 3: Expressions and Equations		75	–	110
Topic 4: Pythagoras and Trigonometry		111	–	145
Topic 5: Fractions, Decimals, Percentages, Ratios		146	–	178
Topic 6: Transformations, Shapes, Areas and Volumes		179	–	204
Topic 7: Analysing Statistics	SAMPLE TOPIC	205	–	248
Topic 8: Further Number Techniques	<i>These topics are in both parts of the course to provide flexibility.</i>	249	–	282
Topic 9: Graphs		283	–	304
Topic 10: Measuring Likelihood		335	–	336
ANSWERS		337	–	360



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THE SPOTON GUIDE PART TWO (Y11)

Topic 8: Further Number Techniques	<i>These topics are in both parts of the course to provide flexibility.</i>	5	–	38
Topic 9: Graphs		39	–	60
Topic 10: Measuring Likelihood		61	–	92
Topic 11: Arcs, Areas, Volumes and Formulae		93	–	119
Topic 12: Algebra – towards A-Level	SAMPLE TOPIC	120	–	170
Topic 13: Graphs and Functions		171	–	218
Topic 14: Further Geometric Techniques		219	–	251
Topic 15: 3D-Trigonometry and Circle Theorems		252	–	290

The emphasis here is on non-calculator skills, with a particular stress on mental agility. The Teachers' Guide for each topic (in the Resource Pack) includes a list of items that could be used as mental/oral starters and the techniques taught/reviewed here should be repeated regularly over the weeks following their introduction. For details of Teachers' Resource Packs, see the next page.

The course should start with Topic 1. This contains the number techniques that will be assumed thereafter throughout the course.
The rest of the topics are independent and can be done in any order.
Any techniques required within a topic that are taught elsewhere, will be repeated at the point where they are required.



**Order one half price copy of Y10 Spoton and Y11 Spoton
using the Special Offer form
which can be downloaded
from the website
www.mathsisjugglers.co.uk**

Cross-Topic Information

The emphasis here is on non-calculator skills, with a particular stress on mental agility. Where applicable, sections within the topics open with items that can be used as mental/oral starters and the techniques taught/reviewed here should be repeated regularly over the weeks following their introduction.

Each lesson should start with a brief mental/oral starter. As well as developing mental arithmetic expertise, it provides a positive start to the lesson.

Suggestions for these are given on the topic pages that follow.

The aim is to develop a learning ethos where:

- mental techniques are a first resort
- pencil-and paper techniques are used routinely
- standard arithmetic techniques are used on a regular basis
- non-standard arithmetic techniques (jottings) are acceptable provided they are clearly shown
- calculators are used only when the calculations become complex
- calculator functions are understood and used effectively

Calculators to be used should have, as a minimum, the following functions:

+ - x ÷ x^2 \sqrt{x} memory brackets
 x^y or y^x $x^{1/y}$ or $\sqrt[x]{}$ sin cos tan

Students who will take the Intermediate Tier at GCSE will have a wide spread of abilities. It is not expected that every student should do every exercise.

The better students will not need to do the easier exercises. For these students, a brief resumé of a technique, done as an oral starter, is all that is required.

The less able students are not expected to do the extension exercises – but they should be allowed to tackle them, if they feel able to do so.

The course should start with Topic 1. This topic contains the number techniques that will be assumed thereafter throughout the course.

The rest of the topics are independent and can be done in any order. Any techniques required within a topic that are taught elsewhere will be repeated at the point where they are required.



The Y10 and Y11 Spoton Guides deliver both the National Curriculum and the linear GCSE syllabuses for AQA, Edexcel and OCR.

However, the topics have been ordered in such a way as to make the material accessible to teachers and students following one of the modular GCSE syllabuses. To facilitate this, two of the topics are in both the Y10 text and the Y11 text. The topic contents of the whole course is listed on the contents page of both texts.

Topic 7: Analysing Statistics

Printing List

None

Page in text

–

Pack page

–

This topic develops/reviews the non-IT techniques required for doing statistical based coursework pieces.

By the end of the topic, each student will have a good understanding of what is required to produce a good piece of coursework and should be ready to produce a piece of their own.

Suggested mental/oral topics

Finding mean, mode and median of simple data sets (like D2.1)
as well as
The mental arithmetic techniques listed for Topics 1 and 5 (mix and match)

Direct teaching points

Section 5: For continuous data, the intermediate GCSE syllabuses state that students should be able to draw frequency diagrams with equal intervals. The term ‘histogram’ is not used. Section 5 is used here as an introduction to working with continuous data. It will lead on to the work in **Section 14** which deals with histograms properly.

Section 6: This is an excellent task for discussion within small groups, regardless of the ability of the class.

When students are referred to their teacher for answers

E1.4: The changeover to unleaded petrol ANSWERS p208

It is important that students learn for themselves just how easily the same data can be used to represent different viewpoints.

1. & 2. Look for a clear set of facts backed up by figures quoted from the multiple bar chart.

E5.4: Bat survey ANSWERS p219

The expected answer is that since Table A has an estimated mean of 6.804g and Table B has an estimated mean of 7.718g, and as the pipistrelle is the smallest bat, it could be assumed that the pipistrelle is the bat in Table A. Note also that there are 97 bats referred to in Table A and 102 in Table B, so either the naturalists meant "around 100" when they said 100, or they have some of the data mixed up.

D6.1: A question of bias ANSWERS p220

4. Check that the question designed on fox-hunting is a neutral question with at least three possible responses.
5. Check that the question designed on whether PE ought to be compulsory is a neutral question with at least three possible responses.

E13.7: Using random numbers to make a sample estimate ANSWERS p244

This is based on a problem in an A-Level Geography book. In all the problems I found in textbooks that used random numbers, none of them actually explained how to use random number tables! Here, not only are the techniques explained but three variations on this method of sampling are tried.

Task 1: The majority of points specified by this method are not on this grid.

Task 2: Rough Grazing 16% Woodland 14% Orchard 8% Other 62%

Task 3: Rough Grazing 9%C & 7% S Woodland 9%C & 5% S

Miscellaneous word problems for use in mental/oral sessions

Exercise 1

1. List the numbers between 63 and 98 which are divisible by 5.
2. Double 45 + treble 7
3. Half of 15 + double 15
4. Paul has 6 red smarties, 3 blue smarties and 9 yellow smarties. He eats 5 smarties. How many does he have left ?
5. 4 twelves + 3 sixes
6. There are 31 days in October. What date will it be 12 days after 26th October ?
7. The world's first underground railway was opened in London in 1863. How many years ago was that ?
8. What is 27 less than 346 ?
9. There are 12 in a dozen. How many are there in $3\frac{1}{2}$ dozen ?
10. In old measurements, 1 stone = 14 pounds. How many pounds are there in 5 stones 4 pounds ?

Exercise 2

1. Double 17 and double $15\frac{1}{2}$
2. What is the smallest multiple of 9 above 85 ?
3. There are 7 players in a netball team. How many complete teams can be made from 61 players ?
4. In old measurements, there were 12 inches in 1 foot. How many inches are there in 3 feet 11 inches ?
5. Write 67 inches in feet and inches.
6. How many twentieths are there in two fifths ?
7. Oscar scored 57% in the text. Abdul scored 18% less than Oscar. Erroll scored 23% more than Abdul. What percentage mark did Erroll get ?
8. Grannie is 72. Mum is half as old as Grannie. Mary is half as old as Mum. How old is Mary ?
9. There are 347 boys and 329 girls in our school. How many students are there in the school ?
10. I had a piece of ribbon 4 m 35 cm long. I cut off two pieces, each 1 m 20 cm long. How long is the piece of ribbon that I have left ?

Exercise 3

1. The first public train service was between Stockton and Darlington. It opened in 1825. How many years ago was that.
2. A number squared is 196. What is the number ?
3. Zara was 27 years old in 1981. In what year was Zara born ?
4. The recipe calls for 8 ounces of icing sugar. Which packet should I buy : 100g, 200g or 300g ?
5. What do you add to 347 to make 420 ?
6. What fraction is halfway between $\frac{1}{5}$ and $\frac{1}{10}$?
7. 9×125
8. There are 12 marbles in a full box. How many boxes can I fill with 159 marbles ?
9. A pyramid of snooker balls has 1 in the top layer, 4 in the second layer and 9 in the third layer. How many balls are there in the fifth layer ?
10. Double 48 + half of 48

Exercise 4

1. What is the value of 2 to the power of 5 ?
2. There are 8 furlongs in one mile. In a 2 mile 5 furlong race, a jockey falls 7 furlongs from the finish. How far is he from the start ?
3. Yesterday 7 pupils in 11JY were absent. This is 20% of the tutor group. How many students are there in the tutor group ?
4. Tarporley is approximately 15 miles from Chester. How much is that approximately in kilometres ?
5. Sam was born in 1942. He will get a pension when he is 65. In what year will he first get his pension ?
6. In Scrabble, my score stands at 278. I then put down a word with a value of 16 points, which also earns a treble word score. What is my score after that ?
7. Work out the value of 6 cubed.
8. Before 1971, there were 12 pennies in one shilling and 20 shillings in one pound. How many pennies were there in one pound ?
9. The Mallard holds the world record speed for steam trains, 127 miles per hour. The record was set in 1938. How many years ago was that ?
10. What is 25 less than 462 ?

Analysing Statistics

REVISION



Name :

Do the answers to this revision sheet in your exercise book or on paper – NOT ON THIS SHEET. Check your answers using the answers on the reverse of this sheet.

KEEP THIS SHEET SOMEWHERE SAFE.
USE IT AGAIN TO REVISE FOR EXAMS.

1. Say whether each set of data is qualitative (QL) or quantitative (QN)

- A: suit of playing card
B: score of playing card
C: colour of playing card

2. Say whether each set of data is discrete (D) or continuous (C)

- P: body temperature of each student in the class
Q: number in each tutor group
R: age of each student

3. Find the mode, median, mean and range of each set of data:

- (a) 32, 31, 29, 34
(b) 7 9 8 8 13 9

4. 72, 74, 74, 74, 75, 76, 77, 77, 77, 77, 81

For this data set, use a calculator to work out:

- (a) the mean to 2 d.p. (b) the sum of the items in the set

5. Number of runs Katy scored in cricket matches Key $317 = 37$

0	0	3	5	7	7	(6)
1	0	5	6	8	(4)	(4)
2	1	8	9	9	(4)	(4)
3	3	6	6	6	(4)	(4)
4	8				(1)	(1)

6. A newspaper girl keeps a record of the amounts she gets for Christmas tips.

She puts them into this table:

Amount	Frequency
at least less than	
£0	7
£1.00	3
£2.00	8
£3.00	9
£4.00	5
£5.00	3

- (a) Which is the modal group ?

- (b) Extend the table and calculate an estimate of the mean.

- (c) Which is the median group ?

- (d) Give an estimate of the range of amounts.

7. Draw a frequency diagram for the data in question 6.

8. Say whether each of these statements is neutral (N), emotive (E), leading (L) or leading and emotive (L + E):

A: Do schoolchildren watch too much TV ?

B: Is the amount of TV children watch too much about right too little

C: Don't you think it unhealthy that children spend so much time stuck in front of a TV ?

9. (a) Which of these best describes the correlation between the midday temperature and the sales of hot drinks ?

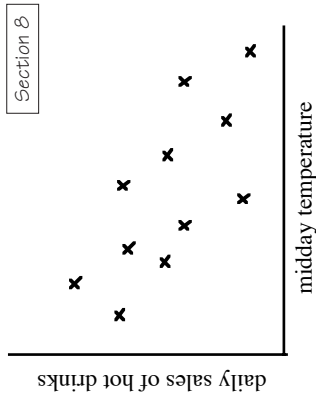
good positive correlation

positive correlation

good negative correlation

negative correlation

(b) Do sales go up or down as the temperature goes up ?



10. Number of absentees each day for a three week period in an office.

5 3 2 0 1 | 2 1 0 2 3 | 6 3 2 1 1

Copy this data list and underneath write the 5-point moving averages.

11. This is a summary of the marks of Pan-Galactic students taking the BioTechnic exam.



Youslas

(a) Copy the table and complete the cumulative frequency column.

(b) Make a cumulative frequency diagram.

Use the cumulative frequency diagram to do the following:

- (c) Estimate the median mark.

- (d) Estimate the upper and lower quartiles.

- (e) Work out the inter-quartile range.

- (f) Estimate how many candidates got 50 or less.

- (g) 20 candidates failed the exam. Find the pass mark.

Examination marks for BioTechnics	Freq	Cu. Freq
under 20	5	
20 and under 40	13	
40 and under 60	25	
60 and under 80	28	
80 and under 100	9	



Big Edd

12. The same students took the Galactospeak exam.

Their results are shown on this box-and-whisker diagram.



Section 11

(a) Copy the Galactospeak exam box-and-whisker diagram.

Below it, draw a box-and-whisker diagram for the BioTech exam.

(b) Using the box-plots only, say whether each of the following statements is true (T) or false (F).

- A: The middle 50% of results for the Galactospeak exam was more spread out than those for the BioTech exam.
- B: The middle 50% of results for the BioTech exam were fairly evenly spaced about the median.
- C: The median for the Galactospeak exam was higher than the median for the BioTech exam.



13. A survey was done of the amount of pocket money a group of 14 year-olds got.

These are the results:

Pocket money	frequency	frequency density
$0p \leq M < 150p$	24	
$150p \leq M < 250p$	25	
$250p \leq M < 500p$	36	
$500p \leq M < 1000p$	20	

Copy the table and fill in the frequency density column.

14. Draw a histogram to show these results.

Section 14

ANSWERS to TOPIC 7

1. A: QL B: QN C: QL

2. P: C Q: D R: C

3. (a) no mode median = 31.5 range = 5
 (b) mode = 8 and 9 median = 8.5 mean = 9 range = 6

4. (a) 75.82 (b) 834

5. (a) 19 (b) 48 (c) 18 (d) 36

6. (a) £3 – £4 (b) £2.81 (c) £2 – £3 (d) £6

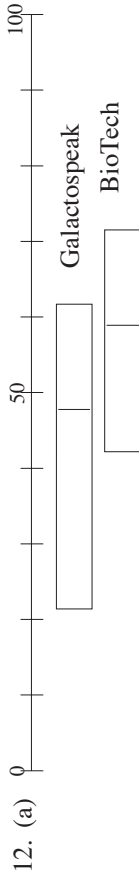
8. A: L B: N C: L + E

9. (a) negative correlation (b) down

10. 5 3 2 0 1 2 1 0 2 3 6 3 2 1 1
 2.2 1.6 1.2 0.8 1.2 1.6 2.4 2.8 3.2 3.0 2.6

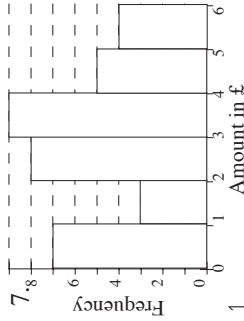
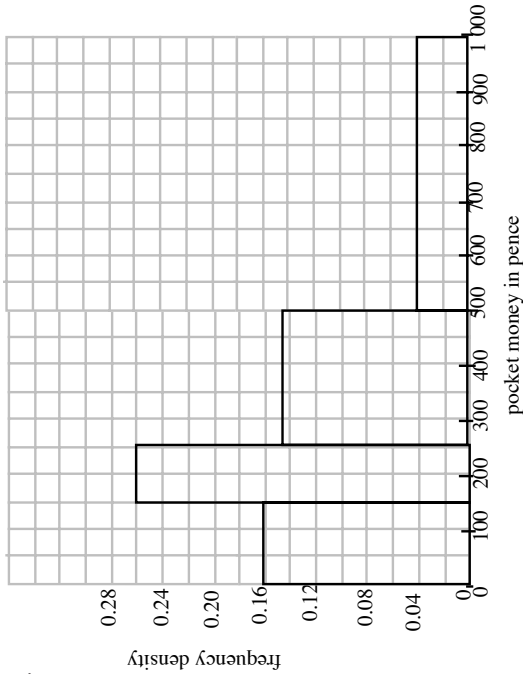
11. (a) 5 18 43 71 80 (c) median \approx 58 (d) LQ \approx 42 UQ \approx 72

(e) \approx 30 (f) \approx 31 (g) \approx 41



(b) A: True B: True C: False

13. Frequency densities are : 0.16, 0.25, 0.144, 0.04



Analysing Statistics ASSESSMENT



Overall mark = %

Name :

Write the answers on this sheet.

1. Find the mode, median, mean and range of each set of data: (8 marks)

(a) 17, 16, 16, 32, 50

mode = median =

mean = range =

(b) 57, 58, 59, 54, 56, 52

mode = median =

mean = range =

2. 125, 125, 126, 128, 128, 128, 129, 131 (2 marks)

mean =

Use statistical calculator functions to work out the mean of this data set.

3. Probability Test Mark Key 317 = 37% Statistics Test Mark

(0)	3	7	(1)
(1)	4	6	(1)
(3)	9	4	(3)
(5)	8	7	(8)
(6)	7	6	(9)
(8)	8	6	(6)
(7)	8	5	(3)

Lowest mark = Highest mark = (8 marks)

Which test was easier?

Explain how you can tell.

How many students took each test? Prob Stats

Work out the median mark for each test. Prob Stats

4. Say whether each of these statements is neutral (N), emotive (E), leading (L) or leading and emotive (L + E): (3 marks)

A: Do you think that experiments on animals ought to be stopped?

B: Using animals in experiments is cruel.

C: Should animals be used to test cosmetics?

5. Say whether each set of data is discrete (D) or continuous (C) (2 marks)

X: numbers of children in families

Y: heights of children in families

6. Which of these best describes the correlation between the science and maths exam marks? (3 marks)

good positive correlation

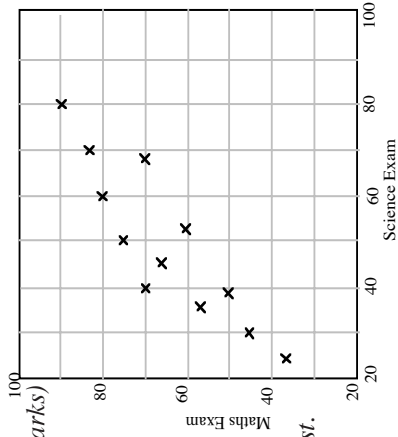
positive correlation

good negative correlation

negative correlation

no apparent correlation

Put a tick on the description you think is best.



7. (a) Draw a line of best fit on the scatter graph in question 6. (5 marks)

(b) Paul did the maths exam, but was ill on the day of the science exam. His maths mark was 75%.

Estimate his science mark from the graph.

Show on the graph how you do it.

estimated science mark%

8. A height survey was done of the boys at the Klaxon Youth Club.

Height in cm	Tally	Frequency
$120 \leq H < 130$		
$130 \leq H < 140$		
$140 \leq H < 150$		
$150 \leq H < 160$		
$160 \leq H < 170$		
$170 \leq H < 180$		

When the survey was done, three boys were not there.

Hashmedi is 170 cm.

Kris is 142 cm

Alan is 130 cm

(a) Add the three missing heights to the table.

(b) Complete the table so that you can estimate the mean height to 1 d.p. Show all working.

Estimated mean height =

(c) What is the modal class of heights

(d) What is the median class of heights

(e) Estimate the range of the heights

(8 marks)

9. A researcher looks up the ages of the first hundred people to die in the parish of Holly in 1880.

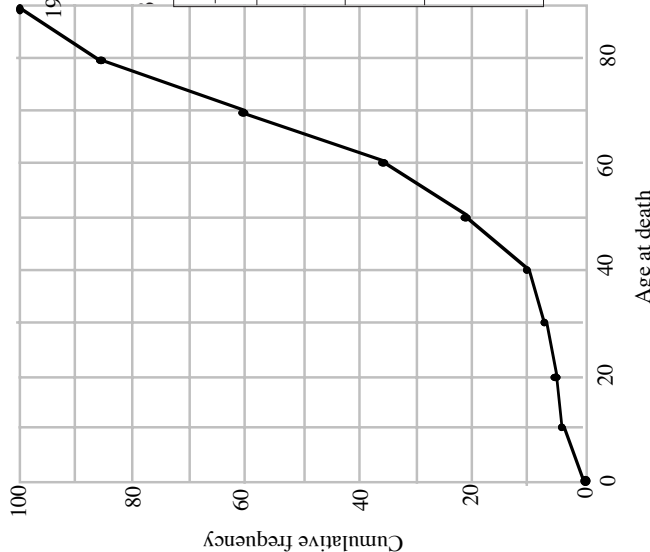
This table shows the results.

Complete the last column of the table to show the cumulative frequency. (2 marks)

The cumulative frequency diagram below shows the ages of the first hundred people to die in Holly in 1880.

Age	Freq	Cu. Freq
under 10	14	
10 and under 20	10	
20 and under 30	9	
30 and under 40	13	
40 and under 50	21	
50 and under 60	24	
60 and under 70	7	
70 and under 80	2	
80 and under 90	0	

10. On the same grid, draw a cumulative frequency diagram for 1880. (4 marks)



Use the graph to complete this table:

	1880	1980
Median		
Lower quartile		
Upper quartile		
Inter-quartile range		

(8 marks)

11. Write down one difference between the ages at death in 1880 compared to 1980.

.....

(2 marks)

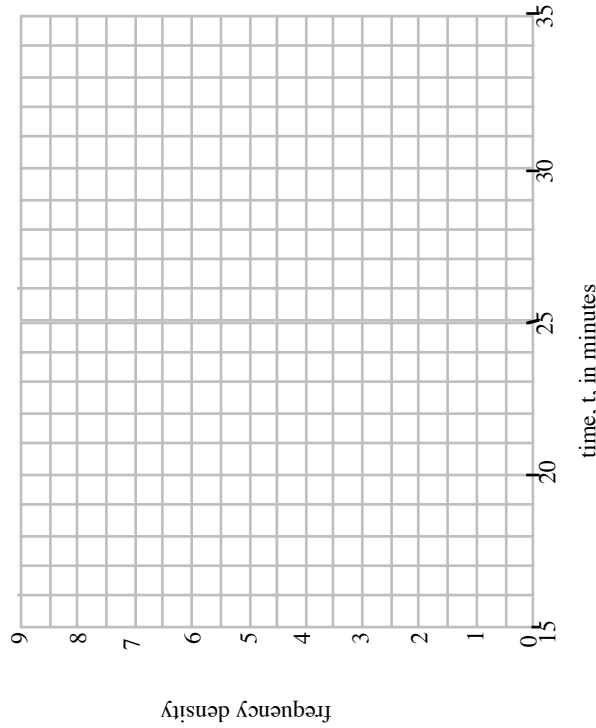
12. Daniel has a paper round. Each day he records how long it takes him to complete his round. His results are shown below.

Time, t, in minutes	No. of days	Frequency density
$15 \leq t < 20$	5
$20 \leq t < 22$	4
$22 \leq t < 24$	8
$24 \leq t < 28$	16
$28 \leq t < 30$	14
$30 \leq t < 35$	10

Fill in the frequency density column. (6 marks)

13. Draw a histogram of the times Daniel takes to complete his paper round.

Space has been provided on the table for Q12, for the working out that needs to be done before the histogram can be drawn.



(6 marks)

OVERALL MARK = / 67

Analysing Statistics

ASSESSMENT ANSWERS

Overall mark = %

Write the answers on this sheet.

1. Find the mode, median, mean and range of each set of data: (8 marks)

(a) 17, 16, 16, 32, 50

mode = ...16... median = ..26.2...

mean = ...17... range = ..34...

(b) 57, 58, 59, 54, 56, 52

mode = no mode median = ..56.5

mean = ...56... range = ..7

2. 125, 125, 126, 128, 128, 128, 129, 131 mean = (2 marks)

Use statistical calculator functions to work out the mean of this data set.

3. Probability Test Mark Key 317 = 37% Statistics Test Mark

(0)	3	7	(1)
(0)	4	6	(1)
(3)	9	5	(3)
(5)	8	4	5
(6)	7	4	3
(6)	6	5	4
(8)	8	6	4
(7)	8	5	4

Lowest mark = ..37.. Highest mark = ..98.. (8 marks)

Which test was easier? ..Probability... (3 marks)

Explain how you can tell. ...the test marks were higher overall.....

How many students took each test? Prob ..29.. Stats ..31..

Work out the median mark for each test. Prob ..80.. Stats ..73..

4. Say whether each of these statements is neutral (N), emotive (E), leading (L) or leading and emotive (L + E): (3 marks)

A: Do you think that experiments on animals ought to be stopped? ...L.....

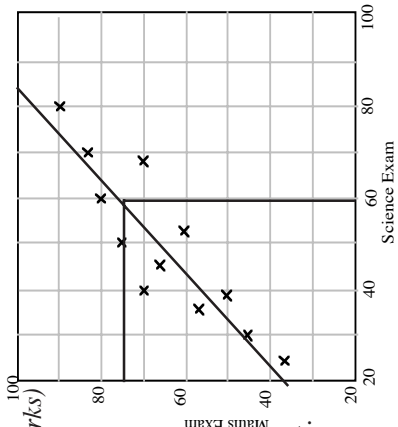
B: Using animals in experiments is cruel. ...E.....

C: Should animals be used to test cosmetics? ...N.....

5. Say whether each set of data is discrete (D) or continuous (C) (2 marks)

X: numbers of children in families ..D..

Y: heights of children in families ..C..



6. Which of these best describes the correlation between the science and maths exam marks? (3 marks)

good positive correlation
 positive correlation
 good negative correlation
 negative correlation
 no apparent correlation

Put a tick on the description you think is best.

7. (a) Draw a line of best fit on the scatter graph in question 6. (5 marks)

(b) Paul did the maths exam, but was ill on the day of the science exam. His maths mark was 75%.

Estimate his science mark from the graph. Show on the graph how you do it.

estimated science mark 60...%

8. A height survey was done of the boys at the Klaxon Youth Club.

Height in cm	Tally	Frequency	Midpoint	M x F
120 ≤ H < 130		4	125	500
130 ≤ H < 140		9	135	1215
140 ≤ H < 150		11	145	1595
150 ≤ H < 160		9	155	1395
160 ≤ H < 170		6	165	990
170 ≤ H < 180		3	175	525
		42		6220

When the survey was done, three boys were not there.
 Hashim is 170 cm. Kris is 142 cm. Alan is 130 cm.

(a) Add the three missing heights to the table. (1 mark)

(b) Complete the table so that you can estimate the mean height to 1 d.p. Show all working. (2 M, 2A)

Estimated mean height = ..148.1 cm.....

(c) What is the modal class of heights ...140-150 cm... (1 mark)

(d) What is the median class of heights ...140-150 cm... (1 mark)

(e) Estimate the range of the heights ...60 cm... (1 mark)

(8 marks)

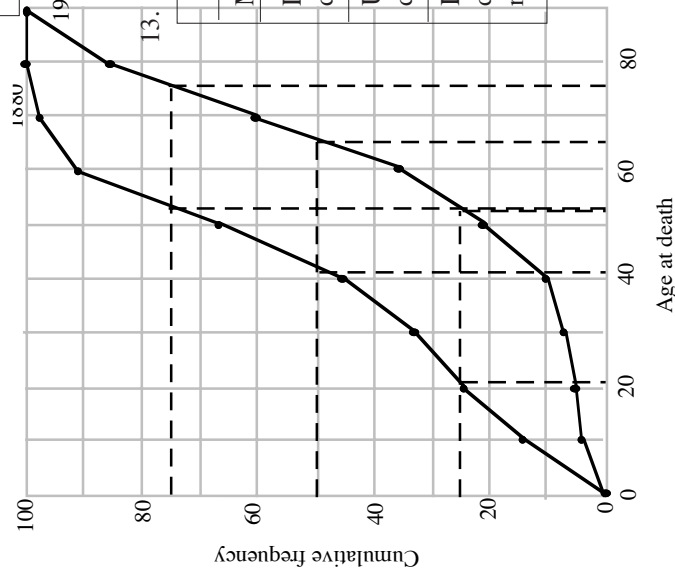
9. A researcher looks up the ages of the first hundred people to die in the parish of Holby in 1880.

This table shows the results.
Complete the last column of the table to show the cumulative frequency. (2 marks)

The cumulative frequency diagram below shows the ages of the first hundred people to die in Holby in 1880.

Age in years at death in 1880	Freq	Cu. Freq
under 10	14	14
10 and under 20	10	24
20 and under 30	9	33
30 and under 40	13	46
40 and under 50	21	67
50 and under 60	24	91
60 and under 70	7	98
70 and under 80	2	100
80 and under 90	0	100

10. On the same grid, draw a cumulative frequency diagram for 1880. (4 marks)



13. Use the graph to complete this table:

	1880	1980
Median	41	65
Lower quartile	21	53
Upper quartile	53	76
Inter-quartile range	32	23

Accept answers in the region of those given in this table. (8 marks)

11. Write down one difference between the ages at death in 1880 compared to 1980.

..... People died younger in 1880.....
 (2 marks)

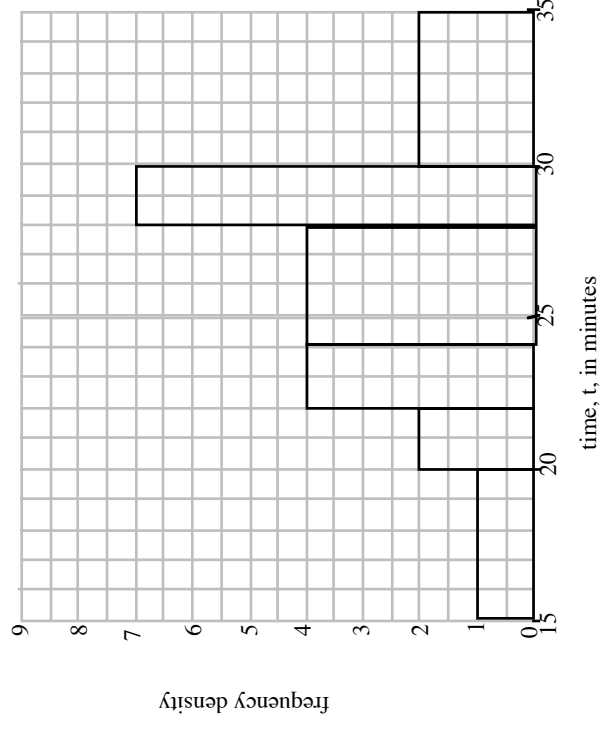
12. Daniel has a paper round. Each day he records how long it takes him to complete his round. His results are shown below.

Time, t, in minutes	No. of days	Frequency density	width	Midpoint
$15 \leq t < 20$	51....	5	17.5
$20 \leq t < 22$	42....	2	21
$22 \leq t < 24$	84....	2	23
$24 \leq t < 28$	164....	4	26
$28 \leq t < 30$	147....	14	29
$30 \leq t < 35$	102....	10	32.5
	57			Working for Q13.

Fill in the frequency density column. (6 marks)

13. Draw a histogram of the times Daniel takes to complete his paper round.

Space has been provided on the table for Q12, for the working out that needs to be done before the histogram can be drawn.



OVERALL MARK = / 67