

ISBN-10: 1-874428-70-0

ISBN-13: 978-1-874428-70-1

Sample topic from the Y9 Optymistic Guide to the National Curriculum

You have permission to print off this topic
and try it with your students.

This document contains the complete topic (p01-27) and answers (p28-30)

Teachers' notes, worksheets, Star Challenge answers, revision and assessment
for this topic can be downloaded and printed off in the
Y9 Optymistic Resource Sample document.

ENJOYMENT

The 'Maths is ...' Jugglers

Knowledge

Skills

Understanding



Optymistic

Patterns and Rules

Most students should start at the beginning of the topic. However, they should be encouraged to attempt Levels 7&8 material if they have the time.

More able students should omit the work at Level 4 and start the topic at the first section which is at Level 5.

Even able Y9 students will benefit from using the Level 5 material as a last review of these topics, before going on to the Higher Level Topics at GCSE (Levels 6-10) in Y10. All Y9 students should at least do the High Level Star Challenges at Level 5.

				§9 Multiplying out two brackets
			§8 Simultaneous equations	
		§7 Systematic equation solving		
		§6 Solving equations		
		§5 Shape patterns		
		§4 Algebra rules, OK ?		
		§3 Machines and rules		
		§2 Using letters for numbers		
§1 Rules in words				
Level 4	Level 5	Level 6	Level 7	Level 8

National Curriculum CONTENTS AND LEVELS

Worksheets are required for:

p17: *10H: Hexagon solution patterns

p21: *11H: Find the fours

p22: *12H- Equation crossnumber



THE OPTYMISTIC GUIDE

to Basics	pp 005 – 034	
Bar Charts and Beyond	pp 035 – 064	
Working with Numbers	pp 065 – 084	
Areas, Volumes and Formulae	pp 085 – 122	
Probability	pp 123 – 144	
Fractions, Decimals and Percentages	pp 145 – 168	
The Geometry of Angle and Shape	pp 169 – 206	
Patterns and Rules	SAMPLE TOPIC	pp 207 – 234
Coordinates and Graphs	pp 235 – 278	
Practical and Computational Measurement	pp 279 – 310	
Extending Algebraic Techniques (must go after Patterns & Rules)	pp 311 – 327	
ANSWERS	pp 328 – 352	

Teachers may change the order of the topics.
However, the topics in this text are in order of difficulty.

For each topic in Y9, we suggest that the date of the topic test is announced in advance. Students then do as much of the topic as they can before that date.

Most students should start at the beginning of the topic and work as far as they can through the topic. However, they should be encouraged to attempt Level 7 material if they have the time.

More able students could omit the work at Level 4 and start the topic at the first section at Level 5.

However, any student who wishes to do this should first seek their teacher's advice.

The material in each topic:

- takes 3-4 weeks to complete
- follows a logical development of mathematical ideas
- progresses up through the levels as students work through the topic
- is labelled with its levels

Each student:

- is given a set period of time to do what they can within the topic (and is given the test date)
- is told that the more they can master in each topic the better they will do in the NCTs in May
- is encouraged to take responsibility for what they do within the time allowed
- should mark their own work and seek help if there is anything that they do not understand or cannot get right.
- can earn stars by tackling the High Level Star Challenges in each topic
- does a revision sheet and assessment at the end of each topic

Each teacher:

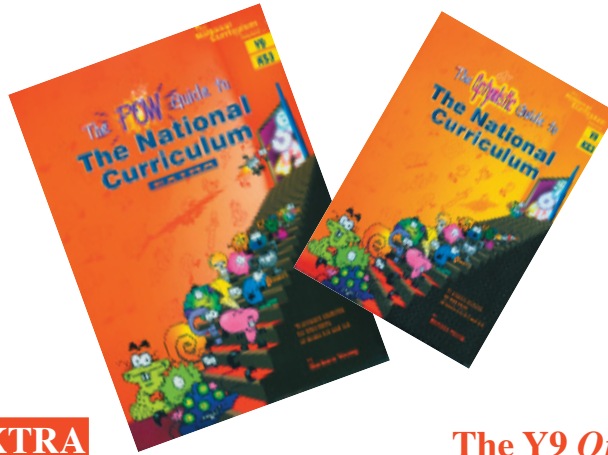
- can work through the topic in the order presented
- can reorder the material, and/or decide what is suitable for individual students using the CONTENTS and LEVELS chart which is at the front of each topic
- can incorporate material from elsewhere that has worked well for them
- **can class-teach some material but will also have time to give plenty of individual help to students**
- at the end of each topic will need about an hour to mark the assessment for each class

The two texts that give to Y9 students of all abilities:

- a totally different approach to mathematics
- motivation
- a well-deserved sense of achievement
- ENJOYMENT



Pow



Optymistic

The Y9 Pow Guide EXTRA

This A4 text delivers levels 3-6 through 9 topics

The Y9 Optymistic Guide

This A5 text delivers levels 4-8 through 11 topics

by Barbara Young

These texts were developed by Barbara in the classroom over several years, with the enthusiastic help of her students, and then trialled in a wide variety of schools.

All students, even those in the lower sets, are extremely well motivated working with this material.

Each topic progresses steadily up the levels and is labelled clearly with the levels.

- **For each topic, choose the best starting point(s) for your students.**
- **Tell them how long they will be allowed to spend on that topic.**
- **See how far they can progress up the levels in that time.**

Better students are not held back by slower classmates.

Teachers have time to spend with those who most need their help.

Watch well-motivated students fly !

Watch confidence soar !

Order half price copies of Y9 Optymistic and Y9 Pow using the Special Offer form which can be downloaded from the website www.mathsisjugglers.co.uk

The POW Guide **EXTRA**

An unusual approach to Mathematics for Low Attainers – that works !!

Students in our lower sets said

“We don’t want a course that only does the easy stuff (a ‘dumbo’ course).

We want to do the same work as everyone else.

- So, will you please:
- make the instructions and explanations clearer
 - introduce ideas more slowly and put in extra steps
 - put in extra practice
 - make the work interesting”

SO WE DID !



Our EXTRA texts for LOW ATTAINERS do not follow the route of so many other texts for these students.

Instead of giving them lots of easy material, we took the syllabus and delivered it in such a way that LOW ATTAINERS could understand and cope with the ideas and techniques.

We gave them what they had asked for !!

But we couldn’t have produced these texts had it not been for the active help of the students themselves. We started with the mainstream texts and students really enjoyed finding out what didn’t work for them - but, more importantly, WHY!! They quickly became expert at pinpointing where the material didn’t work for them and helping to find what would work for them.

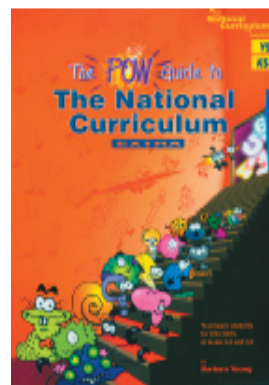
Devising ways that would work took a lot of working/trialling with LOW ATTAINERS, but we ended up with the EXTRA texts – which work exceptionally well, particularly with bottom sets, disaffected students and students at Pupil Referral Units.

This **EXTRA** course :

- has been specially developed for low attainers
- is a version of the Y9 mainstream course
- has lots of EXTRA practice on all techniques
- can be run alongside the mainstream course
- can stand on its own
- is suitable for the lower 50% of the ability range

Each student:

- takes responsibility for his/her own learning
- can decide how much practice (s)he needs to do for each technique
- can try Star Challenges when (s)he feels ready for them
- will be capable of taking the mainstream tests



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The authors firmly believe that all students can tackle the material in the mainstream course.

However, some students need :

- more time to get to grips with the ideas and techniques involved
- lots of EXTRA practice
- one idea at a time introduced step-by-step
- to meet ideas and techniques over and over again

Most students in lower sets are underachieving.

This course aims to raise the level of achievement of these students.

How does the Star Challenge system work ?

The number of stars is a measure of the difficulty of the Challenge.

So, one star denotes a basic fairly easy challenge.

However, students should not be discouraged from attempting any Challenges.

For example, students may earn fewer stars from many two or three star challenges, with partially correct answers.

Students of lower ability can amaze you with what they can achieve, if they really want to tackle problems that you might think beyond them.

A true cautionary tale

Three very low ability students were working together. They had to. Mark was severely dyslexic - he could not read at all. Jason could read very well, but could make no sense of what he was reading. Matthew was very slow at everything. Between them, they could tackle problems.

Jason would read the question. Mark would explain what it was all about. Together the three of them would work out how to solve the problem.

One day they had done all the one star problems. They asked me if they could try the three star problem - The Chest of Drawers. It contained a mixture of fractions and decimals and both cm and mm. My first reaction was to tell them that it was too difficult for them - but I said they could try it, provided they accepted that they might not be able to do it.

About ten minutes later, they brought me the correct answer. I couldn't believe that they had solved such a complex problem. I changed the data (marked below in blue). They were back a few minutes later with the correct answer. Then I made a big mistake. I asked them to tell me how they had worked it out. I couldn't follow their explanation at all !!

From then on I never restricted the choice of Star Challenges of even the least able students.

Star Challenge

The chest of drawers

A chest of drawers 84 cm high has a 42.5 mm top and a 90 mm base. Four identical drawers fit in between, with 22.5 mm between each drawer.

Work out the height of each drawer in cm.

If the 22.5 mm gaps were reduced to 12.5 mm each, what would be the height of each drawer then?

The diagram shows a chest of drawers with a total height of 84 cm. It has a top panel of $4\frac{1}{4}$ cm and a base of 9 cm. There are four drawers in between. The gap between the top panel and the first drawer is $2\frac{1}{4}$ cm. The gap between the second and third drawers is marked with a question mark. The gap between the fourth drawer and the base is $2\frac{1}{4}$ cm. Each drawer has two circular handles.

What do schools do with the stars students earn for Star Challenges ?

One school:

- gives a house point for each star.
- has a commendation system and awards a commendation for so many stars. They also award stars for effort when the books are taken in each fortnight.
- gives a smiley face for 5 stars
- gives a Mars Bar for 10 stars
- gives a copy of one of a group of Chaos posters for 20 stars (very popular with low attainers)
-

It doesn't matter what you do. It is the sense of achievement that is most important.

However, a system that awards something for so many stars, then starts them collecting the next set of stars, means that students keep track of how many stars they have and how many more they need towards the next 'prize' – instead of (possibly invidious) comparisons of the total number of stars each student has.

Patterns and Rules

Level
4

Section 1: Rules in words



In this section you will :

- work with rules for number chains
- find rules for tables of related data
- find and use rules for number machines

DEVELOPMENT

D1.1: Rules for number chains

3 → 5 → 7 → 9 → 11

Rule: Add 2

Write down the first five numbers in each chain:

- 4 → ... → ... → ... → ... Rule : Add 3
- 13 → ... → ... → ... → ... Rule : Take 2
- 2 → ... → ... → ... → ... Rule : Multiply by 2
- 3 → ... → ... → ... → ... Rule : Multiply by 2 and add 1
- 4 → ... → ... → ... → ... Rule : Add 1 and multiply by 2
- 32 → ... → ... → ... → ... Rule : Divide by 2
- 20 → ... → ... → ... → ... Rule : Halve it and add 2
- 4 → ... → ... → ... → ... Rule : Subtract 1 and double it

Find the rule for each chain:

- 10 → 13 → 16 → 19 → 22 Rule :
- 37 → 27 → 17 → 7 → -3 Rule :
- 5 → 10 → 20 → 40 → 80 Rule :
- 100 → 50 → 25 → $12\frac{1}{2}$ → $6\frac{1}{4}$ Rule :
- 1 → 5 → 13 → 29 → 61 Rule :

• Check your answers.

Star Challenge

All correct = 1 star

Write down the first five numbers in each chain:

- 25 → ... → ... → ... → ... Rule : Subtract 4
- 1 → ... → ... → ... → ... Rule : Multiply by 5
- 6 → ... → ... → ... → ... Rule : Double it and take 5

Find the rule for each chain:

- 1 → 3 → 9 → 27 → 81 Rule :
- 3 → 5 → 9 → 17 → 33 Rule :

• Your teacher has the answers to these.

D1.2: From tables to rules

EXAMPLE What is the rule connecting the data in this table ?

number of pencils	cost in pence
1	10
2	20
5	50

Rule:
Cost in pence = 10 x number of pencils

Find the rule in words for each table:

1.

Number of packets	Number of sweets
1	15
2	30
3	45

2.

Number of letters	Postage (in p)
2	40
3	60
5	100

3.

Number of diamonds	Perimeter
1	4
2	8
3	12

4.

Number of lines	Number of dots
1	2
2	3
3	4

• Check your answers.

D1.3: Number machines

Copy and complete:

	Number machine	Table												
1. Add 4	<table border="1" style="margin: auto;"> <tr> <td style="padding: 2px;">In</td> <td style="padding: 2px; text-align: center;">+ 4</td> <td style="padding: 2px;">Out</td> </tr> </table>	In	+ 4	Out	<table border="1" style="margin: auto;"> <tr> <td style="padding: 2px;">In</td> <td style="padding: 2px;">5</td> <td style="padding: 2px;">10</td> <td style="padding: 2px;">12</td> </tr> <tr> <td style="padding: 2px;">Out</td> <td style="padding: 2px;">9</td> <td style="padding: 2px;"></td> <td style="padding: 2px;"></td> </tr> </table>	In	5	10	12	Out	9			
In	+ 4	Out												
In	5	10	12											
Out	9													
2. Subtract 3	<table border="1" style="margin: auto;"> <tr> <td style="padding: 2px;">In</td> <td style="padding: 2px;"></td> <td style="padding: 2px;">Out</td> </tr> </table>	In		Out	<table border="1" style="margin: auto;"> <tr> <td style="padding: 2px;">In</td> <td style="padding: 2px;">4</td> <td style="padding: 2px;">8</td> <td style="padding: 2px;">10</td> </tr> <tr> <td style="padding: 2px;">Out</td> <td style="padding: 2px;"></td> <td style="padding: 2px;"></td> <td style="padding: 2px;"></td> </tr> </table>	In	4	8	10	Out				
In		Out												
In	4	8	10											
Out														
3. Multiply by 3	<table border="1" style="margin: auto;"> <tr> <td style="padding: 2px;">In</td> <td style="padding: 2px;"></td> <td style="padding: 2px;">Out</td> </tr> </table>	In		Out	<table border="1" style="margin: auto;"> <tr> <td style="padding: 2px;">In</td> <td style="padding: 2px;">2</td> <td style="padding: 2px;">5</td> <td style="padding: 2px;">7</td> </tr> <tr> <td style="padding: 2px;">Out</td> <td style="padding: 2px;"></td> <td style="padding: 2px;"></td> <td style="padding: 2px;"></td> </tr> </table>	In	2	5	7	Out				
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4.	<table border="1" style="margin: auto;"> <tr> <td style="padding: 2px;">In</td> <td style="padding: 2px; text-align: center;">x 5</td> <td style="padding: 2px;">Out</td> </tr> </table>	In	x 5	Out	<table border="1" style="margin: auto;"> <tr> <td style="padding: 2px;">In</td> <td style="padding: 2px;">2</td> <td style="padding: 2px;">4</td> <td style="padding: 2px;">6</td> </tr> <tr> <td style="padding: 2px;">Out</td> <td style="padding: 2px;"></td> <td style="padding: 2px;"></td> <td style="padding: 2px;"></td> </tr> </table>	In	2	4	6	Out				
In	x 5	Out												
In	2	4	6											
Out														
5. Square it	<table border="1" style="margin: auto;"> <tr> <td style="padding: 2px;">In</td> <td style="padding: 2px; text-align: center;">x itself</td> <td style="padding: 2px;">Out</td> </tr> </table>	In	x itself	Out	<table border="1" style="margin: auto;"> <tr> <td style="padding: 2px;">In</td> <td style="padding: 2px;">2</td> <td style="padding: 2px;">5</td> <td style="padding: 2px;">7</td> </tr> <tr> <td style="padding: 2px;">Out</td> <td style="padding: 2px;"></td> <td style="padding: 2px;"></td> <td style="padding: 2px;"></td> </tr> </table>	In	2	5	7	Out				
In	x itself	Out												
In	2	5	7											
Out														
6. Double and add 1	<table border="1" style="margin: auto;"> <tr> <td style="padding: 2px;">In</td> <td style="padding: 2px; text-align: center;">x 2</td> <td style="padding: 2px; text-align: center;">+ 1</td> <td style="padding: 2px;">Out</td> </tr> </table>	In	x 2	+ 1	Out	<table border="1" style="margin: auto;"> <tr> <td style="padding: 2px;">In</td> <td style="padding: 2px;">2</td> <td style="padding: 2px;">4</td> <td style="padding: 2px;">5</td> </tr> <tr> <td style="padding: 2px;">Out</td> <td style="padding: 2px;"></td> <td style="padding: 2px;"></td> <td style="padding: 2px;"></td> </tr> </table>	In	2	4	5	Out			
In	x 2	+ 1	Out											
In	2	4	5											
Out														
7. Add 1 and double it	<table border="1" style="margin: auto;"> <tr> <td style="padding: 2px;">In</td> <td style="padding: 2px; text-align: center;">+ 1</td> <td style="padding: 2px; text-align: center;">x 2</td> <td style="padding: 2px;">Out</td> </tr> </table>	In	+ 1	x 2	Out	<table border="1" style="margin: auto;"> <tr> <td style="padding: 2px;">In</td> <td style="padding: 2px;">2</td> <td style="padding: 2px;">4</td> <td style="padding: 2px;">5</td> </tr> <tr> <td style="padding: 2px;">Out</td> <td style="padding: 2px;"></td> <td style="padding: 2px;"></td> <td style="padding: 2px;"></td> </tr> </table>	In	2	4	5	Out			
In	+ 1	x 2	Out											
In	2	4	5											
Out														

8. Does the rule 'double it and add 1' give the same results as 'add 1 and double it' ?

• Check your answers.

Section 2: Using letters for numbers Level 5

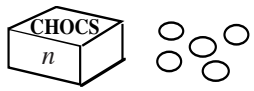
In this section you will:

- use letters to represent unknown numbers
- review the meanings of various algebraic expressions


DEVELOPMENT


D2.1: Letters for unknown numbers

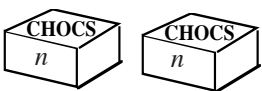
EXAMPLE There are n chocolates in a box.
 Ellie has 1 box + 5 chocolates
 Write an expression to show how many chocolates Ellie has.
 Answer: Ellie has $n + 5$ chocolates



For questions 1 and 2, write an expression for the number of chocolates:

1. 
 Carol has a full box and 2 extra chocolates.

2. 
 Dave had a full box.
 He has eaten 2 chocolates out of the box.

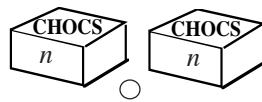
3. 

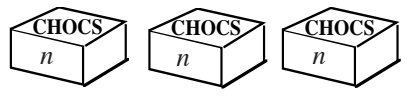
Which two of these expressions are correct for the total number of chocolates ?

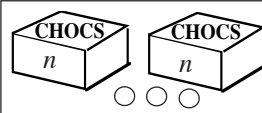
- $n + n$ $2n$ $n + 2$ $n \times n$

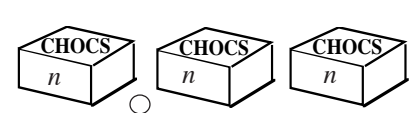
• Check your answers.

Write an expression for the number of chocolates in each case:


4. 

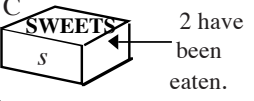
5. 

6. 

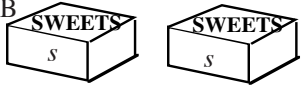
7. 

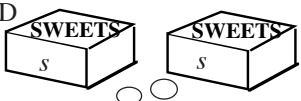
8. Match the pictures with the expressions:

A 

C 

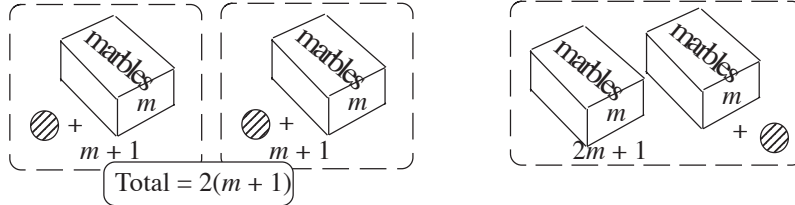
$s + 2$
 $2s$
 $s - 2$
 $2s + 2$

B 

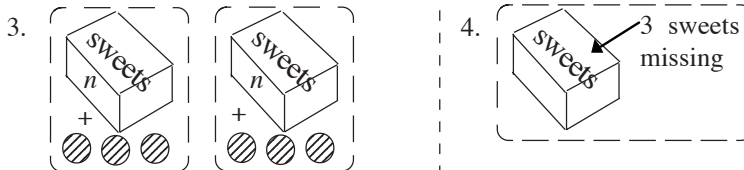
D 

• Check your answers

D2.2: Expressions with brackets



1. Is $2(m + 1)$ the same as $2m + 1$? 2. Is $2(m + 1)$ the same as $2m + 2$?



Copy and complete: 2 lots of

Number of sweets = $2 \times (\dots \dots \dots)$
 $= 2(\dots \dots \dots)$

and $2(\dots \dots \dots) = 2n + \dots$

(a) There are in one box.

(b) There are 2 boxes.
each has 3 sweets missing.

Number of sweets = $2(\dots - \dots)$

• Check your answers.

D2.3: Matching pairs of algebraic cards

$n \div 2$ will always give the same numbers as $\frac{n}{2}$
 They are a matching pair.

Task 1: Find THREE matching pairs from these algebra cards:

$n + n$ n^2 $n + n + 1$ $1 + 2n$ $n \times n$ $2n$

Task 2: Find THREE matching pairs from these algebra cards:

$n + n + n$ $2n + 1$ $2n + n$ $1 + 2n$ $2 + n + 3$ $n + 5$

• Check your answers.

Star Challenge 2H

All correct = 1 star

Task 3: Find THREE matching pairs from these algebra cards:

$\frac{1}{2}n$ $3n$ $n \div 2$ $2n + 3n$ $2n + n$ $5n$

Task 4: Find SIX matching pairs from these algebra cards:

$n + 2$ $2n$ $n + n$ $2n + 1$ $2(n + 1)$ $2n + 2$
 $n + 3$ $2 + n$ $n + n + 1$ $3n + 1$ $2n + n + 1$ $3 + n$

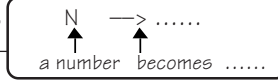
Section 3 : Machines and rules

Level
5

In this section you will work with rules for number machines.

DEVELOPMENT

D3.1: Rules for machines



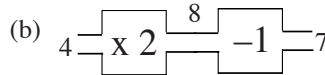
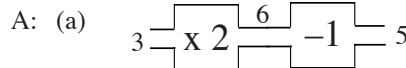
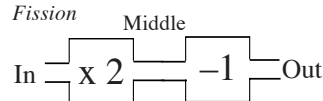
EXAMPLE

Q: (a) Find the 'out' value if you put in 3

(b) Find the 'out' value if you put in 4

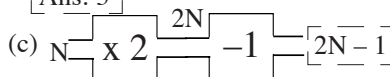
(c) Find the 'out' value if you put in N

(d) What is the rule for this machine?



Ans: 5

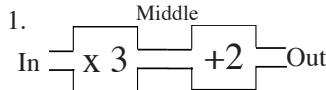
Ans: 7



(d) Rule is $N \rightarrow 2N - 1$



This rule means
"a number becomes twice
the number take away 1"



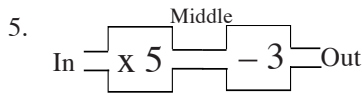
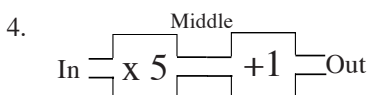
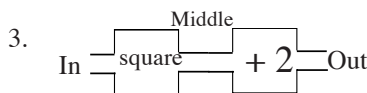
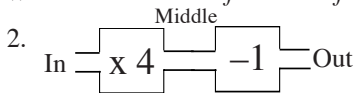
(a) Find the 'out' value if you put in 4

(b) Find the 'out' value if you put in 3

(c) Find the 'out' value if you put in N

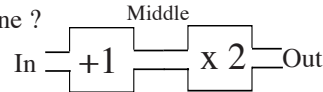
(d) What is the rule for this machine ?

Write down the rule for each of these machines:



6. Which of these is the rule for this machine ?

$N \rightarrow 2(N + 1)$ or $N \rightarrow 2N + 1$



7. Draw machines for each of these rules:

(a) $N \rightarrow 3N + 1$ (b) $N \rightarrow 3(N + 1)$

8. Draw a machine for $N \rightarrow N^2 + 2$

• Check your answers.

D3.2: Machine instructions

Machine 1: $\boxed{\times 2} \rightarrow \boxed{-1}$
 Machine 2: Number $\times 2$ / take away 1
 Machine 3: $2N - 1$

Here are three ways of giving the same instructions.
The three machines are equivalent.

Task 1: Match up the three pairs of equivalent machines:

Machine 1: Number / take away 5
 Machine 2: Number / add 5
 Machine 3: Take number / away from 5
 Machine 4: $N + 5$
 Machine 5: $5 - N$
 Machine 6: $N - 5$

Task 2: Match up the three pairs of equivalent machines:

Machine 1: Number times 2 / add 3
 Machine 2: Number / times 3 add 2
 Machine 3: Number / add 2
 Machine 4: $3N + 2$
 Machine 5: $3(N+2)$
 Machine 6: $2N + 3$

• Check answers.

D3.3: Letters for In and Out

1. $\text{In} \rightarrow \boxed{-4} \rightarrow \text{Out}$ Copy and complete the table for this number machine.

In	Out
10	...
5	...
7	...

2. $a \rightarrow \boxed{-4} \rightarrow b$ The rule for this machine is $b = a - 4$

What is the rule for the machine on the right? $c \rightarrow \boxed{+5} \rightarrow d$

3. $\text{In} \rightarrow \boxed{\times 2} \rightarrow \boxed{-1} \rightarrow \text{Out}$ Copy and complete the table for this number machine.

4. $a \rightarrow \boxed{\times 2} \rightarrow \boxed{-1} \rightarrow b$ The rule for the first machine is $b = 2a - 1$

$p \rightarrow \boxed{\times 3} \rightarrow \boxed{+2} \rightarrow$ What is the rule for the second machine ?

• Check your answers.

Section 4: Algebra rules, OK?

Level
5

In this section you will:

- develop and strengthen your algebraic skills
- practice working with negative numbers

– All individual work
(except for game P3)

PRACTICE

P4.1: Table problems

$S - T$	1 2
T = numbers at top of table	5 4 3
S = numbers at side of table	7 6 5

$S + T$	3 6
5	8 11
4	7 10

Complete each of these tables according to the instructions given by the letters:

1. <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>$2S + T$</td><td>3</td><td>5</td></tr> <tr><td>2</td><td></td><td></td></tr> <tr><td>1</td><td></td><td></td></tr> </table>	$2S + T$	3	5	2			1			2. <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>$S + 2T$</td><td>5</td><td>1</td></tr> <tr><td>3</td><td></td><td></td></tr> <tr><td>4</td><td></td><td></td></tr> </table>	$S + 2T$	5	1	3			4			3. <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>$2S + 2T$</td><td>4</td><td>-1</td></tr> <tr><td>7</td><td></td><td></td></tr> <tr><td>3</td><td></td><td></td></tr> </table>	$2S + 2T$	4	-1	7			3		
$2S + T$	3	5																											
2																													
1																													
$S + 2T$	5	1																											
3																													
4																													
$2S + 2T$	4	-1																											
7																													
3																													
4. <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>$2(S+T)$</td><td>4</td><td>5</td></tr> <tr><td>3</td><td></td><td></td></tr> <tr><td>-1</td><td></td><td></td></tr> </table>	$2(S+T)$	4	5	3			-1			5. <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>$2(S-T)$</td><td>2</td><td>1</td></tr> <tr><td>3</td><td></td><td></td></tr> <tr><td>5</td><td></td><td></td></tr> </table>	$2(S-T)$	2	1	3			5			6. <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>$S - 2T$</td><td>4</td><td>2</td></tr> <tr><td>7</td><td></td><td></td></tr> <tr><td>9</td><td></td><td></td></tr> </table>	$S - 2T$	4	2	7			9		
$2(S+T)$	4	5																											
3																													
-1																													
$2(S-T)$	2	1																											
3																													
5																													
$S - 2T$	4	2																											
7																													
9																													

Complete each of these tables:

7. <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>$S + T$</td><td>1</td><td>4</td></tr> <tr><td>2</td><td></td><td></td></tr> <tr><td>4</td><td></td><td></td></tr> </table>	$S + T$	1	4	2			4			8. <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>$S \div T$</td><td>6</td><td>4</td></tr> <tr><td>12</td><td></td><td></td></tr> <tr><td>24</td><td></td><td></td></tr> </table>	$S \div T$	6	4	12			24			9. <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>$\frac{T}{S}$</td><td>4</td><td>1</td></tr> <tr><td>2</td><td></td><td></td></tr> <tr><td>-1</td><td></td><td></td></tr> </table>	$\frac{T}{S}$	4	1	2			-1		
$S + T$	1	4																											
2																													
4																													
$S \div T$	6	4																											
12																													
24																													
$\frac{T}{S}$	4	1																											
2																													
-1																													
10. <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>$ST + 1$</td><td>2</td><td>-2</td></tr> <tr><td>3</td><td></td><td></td></tr> <tr><td>4</td><td></td><td></td></tr> </table>	$ST + 1$	2	-2	3			4			11. <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>$T^2 + S$</td><td>2</td><td>3</td></tr> <tr><td>3</td><td></td><td></td></tr> <tr><td>-2</td><td></td><td></td></tr> </table>	$T^2 + S$	2	3	3			-2			12. <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>$S^2 + 2T$</td><td>3</td><td>1</td></tr> <tr><td>3</td><td></td><td></td></tr> <tr><td>4</td><td></td><td></td></tr> </table>	$S^2 + 2T$	3	1	3			4		
$ST + 1$	2	-2																											
3																													
4																													
$T^2 + S$	2	3																											
3																													
-2																													
$S^2 + 2T$	3	1																											
3																													
4																													

Star Challenge 3H

Find the rule for each table:

1. <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>?</td><td>4</td><td>3</td></tr> <tr><td>2</td><td>6</td><td>5</td></tr> <tr><td>7</td><td>11</td><td>10</td></tr> </table>	?	4	3	2	6	5	7	11	10	2. <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>?</td><td>5</td><td>2</td></tr> <tr><td>1</td><td>7</td><td>4</td></tr> <tr><td>3</td><td>11</td><td>8</td></tr> </table>	?	5	2	1	7	4	3	11	8	3. <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>?</td><td>2</td><td>3</td></tr> <tr><td>6</td><td>11</td><td>17</td></tr> <tr><td>9</td><td>17</td><td>26</td></tr> </table>	?	2	3	6	11	17	9	17	26
?	4	3																											
2	6	5																											
7	11	10																											
?	5	2																											
1	7	4																											
3	11	8																											
?	2	3																											
6	11	17																											
9	17	26																											
4. <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>?</td><td>3</td><td>-2</td></tr> <tr><td>4</td><td>21</td><td>6</td></tr> <tr><td>1</td><td>12</td><td>-3</td></tr> </table>	?	3	-2	4	21	6	1	12	-3	5. <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>?</td><td>6</td><td>10</td></tr> <tr><td>2</td><td>4</td><td>6</td></tr> <tr><td>1</td><td>7</td><td>11</td></tr> </table>	?	6	10	2	4	6	1	7	11	6. <table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>?</td><td>3</td><td>1</td></tr> <tr><td>2</td><td>7</td><td>5</td></tr> <tr><td>5</td><td>28</td><td>26</td></tr> </table>	?	3	1	2	7	5	5	28	26
?	3	-2																											
4	21	6																											
1	12	-3																											
?	6	10																											
2	4	6																											
1	7	11																											
?	3	1																											
2	7	5																											
5	28	26																											

• Check your answers.

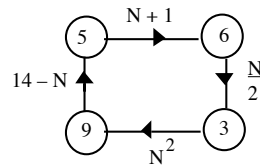
All correct = 1 star

• Your teacher has the answers to these.

P4.2: Number chains

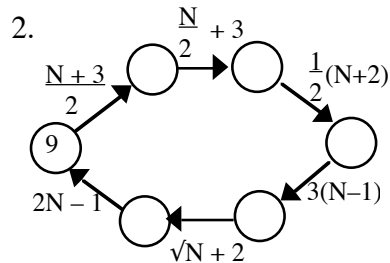
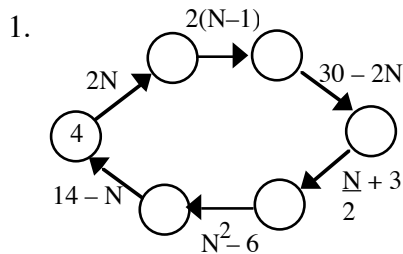
Key to algebraic expressions		
Expression	Meaning	Value when N = 4
$2N$	2 x the number	8
$\frac{1}{2}N$ or $\frac{N}{2}$	half the number	2
$3N + 1$	multiply the number by 3, then add 1	13
$3(N + 1)$	add 1 to the number, then multiply by 3	15
$\frac{N + 1}{2}$	divide the number by 2, then add 1	3
$\frac{N+1}{2}$ or $\frac{1}{2}(N+1)$	add 1, then divide by 2	2.5
$N - 5$	take 5 away from the number	1
$5 - N$	take the number from 5	-1
N^2	square the number	16
$3N^2$	square the number and multiply by 3	48
$(3N)^2$	multiply the number by 3, then square the answer	144
$N(N-1)$	the number x one less than the number	12
$\sqrt{N + 5}$	find the square root of the number, then add 5	7
$\sqrt{N + 5}$	add 5 to the number, then find the square root	3
$\frac{N+2}{N-1}$	(two more than the number) ÷ (one less than the number)	2
$N - 1$		

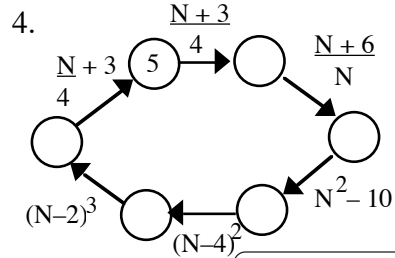
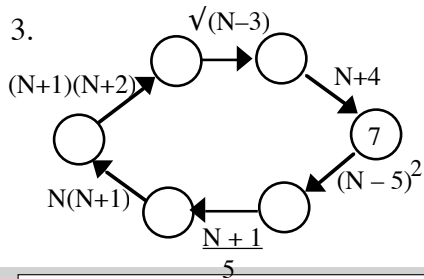
In each chain, each number is made from the number before it, using the rule between the numbers.



For each question:

- find the missing numbers;
- write down the numbers in the chain, in order.

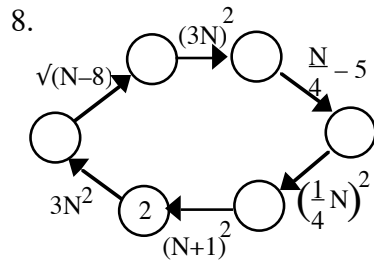
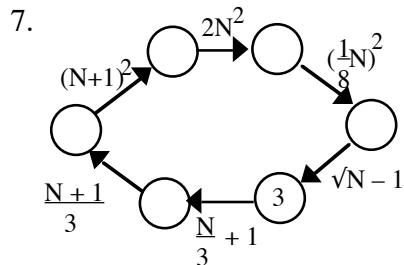
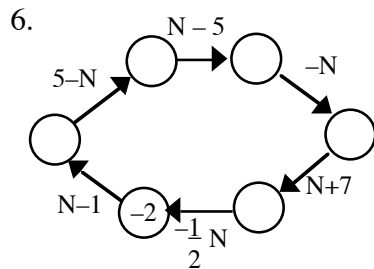
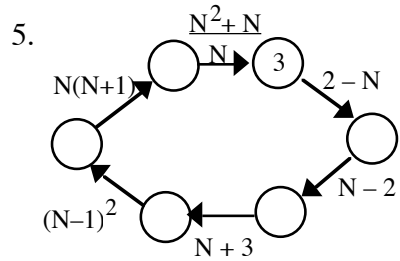




For Q5 – 8

-4 -3 -2 -1 0 1 2 3 4

- (-2) = 2
 + (-2) = -2
 - x - = +
 - x + = -

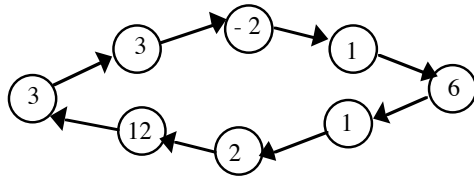


• Check your answers.

Star Challenge 4H

All correct = 1 star

9. Put the rules in the correct places.



RULES

$2N - 3$	$N - 5$	$\frac{1(N+4)}{2}$
$(N + 1)(N + 2)$		$\frac{N+3}{N+1}$
$(N - 5)^2$	$\frac{N-3}{N-9}$	
$3N^2$		

• Your teacher has the answers to these.

P4.3: From A to B exactly

1 special dice (see below)

A game for 2 – 4 people

THIS GIVES PRACTICE BOTH IN USING ALGEBRA AND NEGATIVE NUMBERS.

You will find the board for this game on the next page.

There is an A4 version of this game in the Teachers' Resource Pack.

You will need one counter for each player and a dice, marked 1, 1, 2, 2, 3, 3 (stick labels on an ordinary dice?)

Rules

Each player starts with their counter on the START square. The winner is the first player to get from the START square to the HOME square.

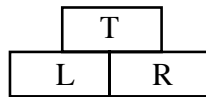
For each move, the player:

- states whether they want a positive or negative number;
- throws the dice;
- puts the number they throw into the expression on their square and finds the value of the expression (if they had stated that they wanted a negative number and shook 1, they put in -1);
- moves the same number of places as the value of the expression – forward is positive and backward is negative.

Star Challenge 5H 5H

All 3 correct = 2 stars
2 correct = 1 star

Building walls to rule

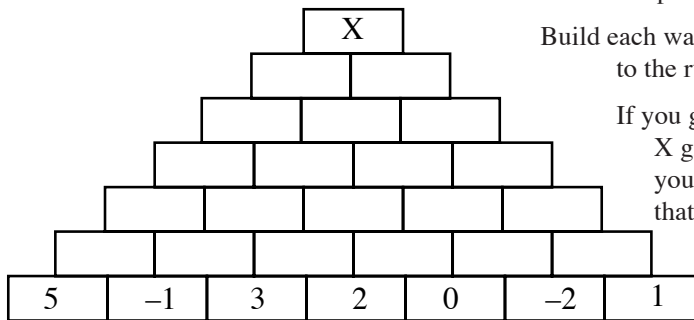


Each brick (T) in the wall is formed from the two bricks below it (L & R). You need to know the rule connecting the three bricks.

Make three copies of the wall.

Build each wall according to the rule given.

If you get the value of X given, then you will know that you are right.



Wall 1 : $T = L + R$ $X = 73$

Wall 2 : $T = L - R$ $X = 29$

Wall 3 : $T = L + 2R$ $X = 173$

- Your teacher has the answers to these walls.



Section 4 – rules for this game are on page 217

From A to B exactly

$-c^2$	$x-10$	$6-n$	$7+p$	$x-8$	$1-2t$	$n-3$	$x+1$	HOME B	Move on 3 places	$m[m-3]$	$2p$
$x+1$	$-3r$	$1-2y$	$4(n-2)$	$x-2$	$f+5$	$2(q+1)$	$s(s+1)$	$\frac{9m}{3}$	$7-r$	$3t-7$	$z-2$
$2m+1$	$\frac{a^2-16}{a+4}$	$t(6-t)$	$p-7$	$6+g$	Move back x places	$t-8$	$3q$	$-3n$	$2y$	$7+x$	$8-s$
$k-6$	$r+2$	$y+1$	$-m^2$	$\frac{x^2-9}{x-3}$	$2t+4$	b^2-10	$n-6$	$t+1$	$q-3$	$x(x-5)$	t^2-20
Move on y places	$k(k-1)$	$2s+1$	$-2p$	$m(m+1)$	$x-5$	n^2	$3t-5$	m^2-20	$t(t+1)$	$\frac{4k}{2}$	$\frac{c^2-1}{c-1}$
$s-10$	$2b$	$3-k$	$3c$	$y+1$	$x-1$	$q+7$	$2-p$	$n-5$	$2x+1$	$1-2n$	$r+3$
c^2	$2p+1$	$m-1$	$a+4$	$b-2$	$3n-8$	$2d+1$	$p-7$	$4+y$	Move back 3 places	START A	$2d-1$
$10-z$	$6-f$	x^2	$s-1$	$e+3$	$2r-1$	$c-2$	$4-2w$	$3n+1$	$10-x$	$2m$	$y+4$



Star Challenge 6H 6H

12-13 correct = 2 stars
10-11 correct = 1 star

Magic Squares

In a **Magic Square**, the sum of every row, every column and every diagonal is the same. This sum is called the **Magic Sum**.

1. Complete each of these magic squares:

5		
	7	
4		

Magic Sum = 21

9		
5		11

Magic Sum = 30

6		5
		9
11		10

Magic Sum = ____

2. There is a connection between the number in the middle of the square and the Magic Sum. What is this connection?

3.

m	$m+4$?
-----	-------	---

 The sum of these three squares is $3m$.
What goes in the third square?

4. The sum of these three squares is $3m$.
What goes in the third square?

$m-2$	m	?
-------	-----	---

Complete each of these magic squares:

5.

m		
	m	
$m+1$		

Magic Sum = $3m$

6.

$m-2$		
$m+3$	m	

Magic Sum = $3m$

7.

$m-1$		
	m	
$m-3$		

Magic Sum = $3m$

8.

	m	$m-1$
$m+3$		

Magic Sum = ?

9.

m		
	$m-1$	
$m-1$		

Magic Sum = $3m-3$

10.

	$m+2$	
m	$m+3$	

Magic Sum = $3m+6$

Complete each of these magic squares:

Hint: look at your answer to Q2.

11.

	m	
$m-p$		$m+p$

12.

$m+2n$		
	m	
$m-n$		

13.

$m+q$		$m-p$
	m	

• Your teacher has the answers to these.

Section 5: Shape patterns

Levels
5 & 6

In this section you will :


- write rules for patterns using algebra (letters instead of words)
- look for patterns in sequences of shapes

DEVELOPMENT

Level
5

D5.1: Rules for shape patterns

1. Copy and complete the table:

		
Lines (L)	1	...
Dots (D)	2	...

2. The rule for this pattern is :

$$\text{Number of dots (D)} = \text{Number of lines(L)} + 1$$

- (a) How many dots will there be with 20 lines ?




$$D = L + 1$$

- (b) One of these expressions is the rule written using algebra.

$$L = D + 1$$

Which expression is the correct one ?

3. Copy and complete the table:

			
shape number(S)	1	2	3
number of matches (M)	4




4. There is a rule connecting the shape number S and the number of matches M. Give the rule in words.
5. Write the rule using algebra. • Check your answers.
6. Work out the number of matches used to make shape number 10. answers.

D5.2: Choose the right formula

A formula is a rule written using algebra instead of words.




In each case, choose the correct formula for the data in the table.

1.

Lines (L)	Spaces (S)
	2
	4
	6




$S = 2L$
$L = 2S$
$S = L + 2$

2.

Squares (S)	Perimeter (P)
	4
	6
	8




$P = 2S$
$P = 2S + 2$
$S = 2P + 2$

3.

Squares (S)	Matches (M)
	4
	7
	10

$M = 3S$
$M = 3S + 1$
$M = 3S + 2$

4.

Pentagons (p)	Distance round (d)
	5
	8
	11

$d = 3p$
$d = 3p + 1$
$d = 3p + 2$

5. Use the formula for question 2. Work out the number of spaces in the circle with 6 lines.

6. Use the formula for question 4.

Work out the distance round the shape with 10 pentagons.

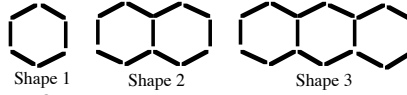
• Check your answers.

Level
6

D5.3: Matchstick patterns

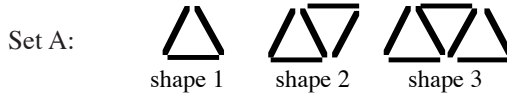
1.

Shape number	1	2	3	4	5
Match sticks	6	11	16	?	?

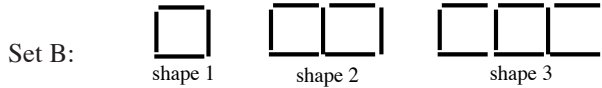


- (a) How many matchsticks will shape 4 have ?
 (b) How many matchsticks will shape 5 have ?
 (c) How do you find the number of matchsticks in the next shape in the pattern?
 (d) How many matchsticks will shape 10 have ?
2. For each set of shapes A, B, C, D and E:

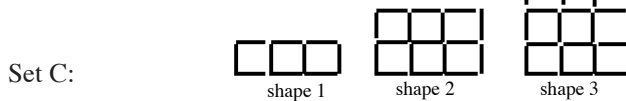
- copy the first three shapes;
- draw shape 4;
- copy and complete the table;
- explain how you find the number of matchsticks in the next shape in the pattern.



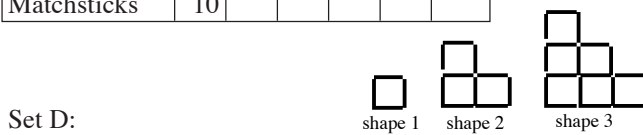
Shape number	1	2	3	4	5	10
Matchsticks	3					



Shape number	1	2	3	4	5	10
Matchsticks	4					



Shape number	1	2	3	4	5	6
Matchsticks	10					



Shape number	1	2	3	4
Matchsticks	4			



Shape number	1	2	3	4
Matchsticks	10			













• Check your answers.
 Patterns and Rules

EXTENSIONS

E5.4: Dotty number patterns

For each of these number patterns:

- copy the first three shapes and write the number of dots underneath
- draw shape 4 and write the number of dots under it
- predict the number of dots in shape 5. Explain why you think it is that number
- draw shape 5 to test your prediction
- write down the sixth and seventh numbers in the sequence.

Pattern	Name of pattern	Shape 1	Shape 2	Shape 3
P	crossnumbers			
Q	square numbers			
R	triangle numbers			
S	diamond numbers			






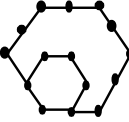
• Check answers.

Star Challenge 7H 7H

3 totally correct = 2 stars
2 totally correct = 1 star

For each of these number patterns:

- copy the first three shapes and write the number of dots underneath
- draw shape 4 and write the number of dots under it
- predict the number of dots in shape 5. Explain why you think it is that number
- draw shape 5 to test your prediction
- write down the sixth and seventh numbers in the sequence.

T	pentagonal numbers			
U	hexagonal numbers			

W Challenge! The first three octagonal numbers are 1,8,21.
Find the next three octagonal numbers.

- Your teacher has the answers to these.

Star Challenge 8H 8H

Row sequences

14-15 marks = 2 stars
12-13 marks = 1 star

2 3 5 7	← 1st row in sequence
5 8 12 9	← 2nd row in sequence

$5 = 2 + 3$ $9 = 7 + 2$

Since 7 is the last number in the row, the number added to it is the first number in the row

a	b
c	

Rule: $c = a + b$

1. Write the first five rows of the sequence above. (3 marks)

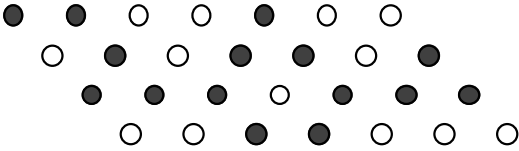
2. Write the first four rows of this next sequence (3 marks)
1st row 3 2 5 1 Rule : $c = a \times b$

2	5	4	7	3
12	13	18	13	7
38	49	44	27	31

(a) What is the rule for this sequence ?
(b) Find the next two rows of the sequence. (3 marks)

4. 1 1 2 1 3
 5 8 7 11 9
Find the next line of this sequence. (2 marks)

5. (a) What is the rule for this sequence ? (2 marks)



(b) Apply the same rule to these dots and find the third row of the sequence. (2 marks)
○ ● ● ○ ○
• Your teacher has the answers to these.

Star Challenge 9H

Sequence challenge

All correct = 1 star

These four sequences are all related.
Find the next two terms in each sequence:

I	II	III	IV	V
O	T	T	F	F
3	3	5	4	4
I	II	III	IV	V

• Your teacher has the answers to these.

Section 6: Solving equations

In this section you will solve equations by inspection.

DEVELOPMENT

D6.1: What is an equation ?

Expressions and equations

$2N + 1$ is an **expression**. For each value of N , the expression has a different value.

$2N + 1 = 7$ is an **equation**.

When $N = 3$, it is a true statement. If N is not 3, it is not a true statement.

$N = 3$ is **the solution** of $2N + 1 = 7$

Expressions and equations can use any letters, not just N .

- $N + 1 = 6$ For what value of N is this true ?
- $a - 2 = 8$ What is the value of a ?
- $3b = 18$ For what value of b is this true ?
- $2p + 1 = 7$ What is the value of p ?

In each question, you have been **solving the equation** or **finding the solution of the equation**.

Solve each equation. Write each answer in the form $N = \dots$

- $N + 2 = 9$
- $2N = 8$
- $3N = 30$
- $N + 5 = 8$
- $2N + 1 = 11$
- $3N = 12$
- $2N - 1 = 19$
- $10N + 1 = 71$

• Check your answers.

Star Challenge 10H

All correct = 1 star

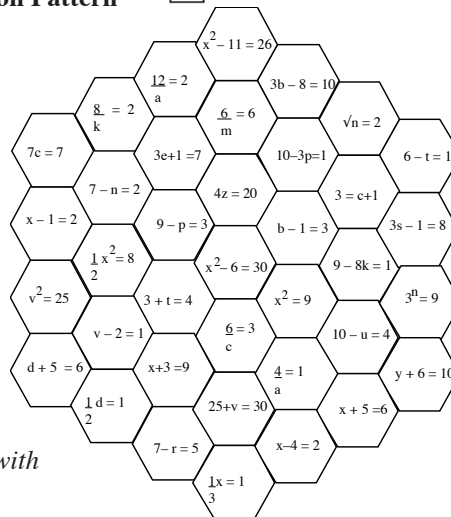
Hexagon Solution Pattern

Solution – colour key

Solution	Colour
1	Red
2	Blue
3	Green
4	Orange
5	Yellow
6	Purple

Solve each equation.

Colour each rectangle with the solution colour.



Section 7: Systematic equation solving

In this section you will review and test your equation solving techniques.

DEVELOPMENT

Level
6

D7.1: Diagnostic testing

You can solve simple equations in your head. But you will need to learn some techniques in order to solve more difficult equations. You have probably met some of these techniques before. To understand the techniques properly, you need to practise them by solving simple equations – the sort you can already solve in your head. Then you will be able to apply these techniques to solving the difficult equations.

Try each set of diagnostic questions. Check your answers at the end of each Test. If you can do the questions confidently **AND can show your working out clearly**, then you do not need to do that set of equations.

If you have any problems with one technique, do that set of equations.

If you would like more practice on one of the techniques, do as much as you need of that set of equations.

HOWEVER, IF YOU WISH, OR YOUR TEACHER ADVISES, YOU CAN JUST DO ALL OF D7.2, D7.4, D7.5, D7.6 WITHOUT DOING THE DIAGNOSTIC TESTS.

Diagnostic Test for D7.2 : Simple equations

Solve these equations. Show all your working clearly.

1. $3p + 2 = 11$ 2. $2c + 40 = 140$ 3. $5t - 3 = 32$

• Check your answers.

Did you show your working as in the answers? If not, you need to do D7.2.

Diagnostic Test for D7.4 : Equations with letters on both sides

Solve these equations. Show all your working clearly.

1. $3 + 4x = 13 - x$ 2. $5m - 6 = 2m + 3$ 3. $6v + 7 = 39 - 2v$

• Check your answers.

Did you show your working as in the answers? If not, you need to do D7.4.

Diagnostic Test for D7.5 : Simple equations with brackets

Solve these equations. Show all your working clearly.

1. $2(3n + 1) = 20$ 2. $3(2p - 5) = 15$ 3. $5(4t + 1) = 65$

• Check your answers.

Did you show your working as in the answers? If not, you need to do D7.5.

Diagnostic Test for D7.6 : Letters on both sides and brackets

Solve these equations. Show all your working clearly.

1. $2(p + 1) = p + 8$ 2. $16 + 3a = 2(a + 11)$ 3. $3(4e - 6) = 2(3e + 6)$

• Check your answers.

Did you show your working as in the answers? If not, you need to do D7.6.

D7.2: The basic technique

$$\begin{array}{r|l} & 2m + 6 = 10 \\ -6 & 2m = 6 \\ \hline +2 & m = 3 \end{array}$$

Write the instructions on *one* side of the equations.



Lubbly

Even though the instructions are only written on one side of the equation, the rule is always: "whatever you do to one side of the equation, you must do the same to the other side."

Copy and complete:

$$\begin{array}{r|l} 1. & 5p + 3 = 18 \\ -3 & = \\ \hline +5 & = \end{array}$$

$$\begin{array}{r|l} 2. & 3k - 1 = 11 \\ +1 & = \\ \hline +3 & = \end{array}$$

$$\begin{array}{r|l} 3. & 4e - 2 = 18 \\ +2 & = \\ \hline +4 & = \end{array}$$

Solve these equations. Set out your working as in questions 1 – 3.

4. $3y + 5 = 14$

5. $7x + 7 = 42$

6. $8b - 3 = 13$

7. $6p - 3 = 15$

8. $9a - 2 = 43$

9. $2p + 1 = 15$

• Check your answers.

PRACTICE

P7.3: Basic technique practice

Solve these equations.

Set out all working as in D7.2.

Do one batch of questions at a time then **CHECK YOUR ANSWERS**.

You may not need to do every batch.

Batch A:

1. $3p + 9 = 24$

2. $6t + 4 = 40$

3. $9h - 7 = 11$

4. $8v - 3 = 21$

5. $8x - 8 = 16$

6. $7c - 5 = 37$

7. $2k + 1 = 11$

8. $7y - 5 = 16$

9. $5s + 3 = 28$

10. $5x - 9 = 16$

11. $5x + 3 = 33$

12. $8j + 6 = 46$

13. $7r + 2 = 79$

14. $2w - 2 = 8$

15. $3q - 3 = 24$

STOP! Have you checked all your answers – and found out where you went wrong?

Batch B:

1. $2d + 7 = 15$

2. $4e + 6 = 50$

3. $5y - 4 = 36$

4. $9n - 2 = 25$

5. $2d - 6 = 10$

6. $2b - 3 = 17$

7. $2b - 5 = 11$

8. $2m - 1 = 29$

9. $5q + 13 = 38$

10. $2x + 3 = 19$

11. $5f + 4 = 49$

12. $9n - 7 = 47$

13. $5t - 6 = 19$

14. $8v - 2 = 6$

15. $3g - 2 = 13$

It is a total waste of your time if you go onto the next batch of questions without checking your answers first! You need to know whether what you are doing is correct!



Optimistic

D7.4: Equations with letters on both sides

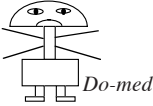
Principles of systematic equation solving

1. Whatever you do to one side of the equation, you must do the same to the other.
2. Aim to get all the letters on one side and all the numbers on the other side.

EXAMPLE

$$\begin{array}{r}
 5 + 4c = 15 - c \\
 + c \quad | \quad 5 + 5c = 15 \\
 - 5 \quad | \quad 5c = 10 \\
 \quad \quad | \quad c = 2
 \end{array}$$

gets all the numbers on one side → ← gets all the letters on one side



Batch C

Solve these equations. Show all your working out.

- | | | |
|-------------------------|------------------------|------------------------|
| 1. $4n + 11 = 3n + 16$ | 2. $4s - 5 = 3s + 2$ | 3. $6v - 7 = 5v - 5$ |
| 4. $8a - 8 = 55 - a$ | 5. $5e - 9 = 47 - 3e$ | 6. $9b - 1 = 97 - 5b$ |
| 7. $8b - 2 = b + 40$ | 8. $8n + 7 = 52 + 3n$ | 9. $5x + 6 = 4x + 8$ |
| 10. $10k + 8 = 9k + 13$ | 11. $6d + 5 = 35 - 4d$ | 12. $10n - 14 = n + 4$ |

• Check your answers. Do you need more practice? If so, do Batch D.

Batch D

Solve these equations. Show all your working out.

- | | | |
|-----------------------|----------------------|------------------------|
| 1. $3a + 1 = a + 9$ | 2. $7y = 3y + 20$ | 3. $12x - 6 = 7x - 1$ |
| 4. $5m = 8 + 3m$ | 5. $5s - 4 = 2s + 5$ | 6. $3n - 7 = n + 1$ |
| 7. $9v - 4 = 5v + 12$ | 8. $9k + 1 = 5k + 9$ | 9. $11w - 51 = 6w - 1$ |
| 10. $4y + 3 = y + 9$ | 11. $3m - 4 = m + 2$ | 12. $8d + 1 = 31 - 2d$ |

• Check your answers.

D7.5: Simple equations with brackets

EXAMPLE

$$\begin{array}{r}
 \text{multiply out} \\
 - 2 \\
 - a
 \end{array}
 \left| \begin{array}{l}
 2(a + 1) = a + 8 \\
 2a + 2 = a + 8 \\
 2a = a + 6 \\
 a = 6
 \end{array} \right.$$

Batch E

Solve these equations. Show all your working out.

- | | | |
|---------------------|---------------------|----------------------|
| 1. $2(a + 8) = 22$ | 4. $2(g + 4) = 30$ | 7. $4(6p - 7) = 92$ |
| 2. $4(m - 2) = 4$ | 5. $9(4c - 1) = 99$ | 8. $4(2d - 5) = 4$ |
| 3. $2(5n - 1) = 18$ | 6. $3(h + 5) = 36$ | 9. $5(4q - 3) = 205$ |

• Check your answers. Do you need more practice? If so, do Batch F.

Batch F

Solve these equations. Show all your working out.

- | | | |
|---------------------|---------------------|---------------------|
| 1. $2(2p + 1) = 18$ | 4. $6(1 + 4x) = 30$ | 7. $3(3m - 1) = 24$ |
| 2. $8(c - 1) = 32$ | 5. $4(3e - 1) = 44$ | 8. $4(2q + 1) = 12$ |
| 3. $2(3m - 4) = 16$ | 6. $9(h - 5) = 18$ | 9. $3(4p - 5) = 21$ |

• Check your answers

D7.6: Letters on both sides and brackets

Batch G Solve these equations. Show all your working out.

1. $3(m-2) = m+2$ 2. $2y = 4(6-y)$ 3. $7(p-3) = 3(7+p)$
 4. $4(v+1) = v+13$ 5. $2(p+1) = 3p$ 6. $5(w+3) = 10(w+1)$
 7. $2(s-2) = s+4$ 8. $2(k+2) = k+6$ 9. $4(2t-3) = (3t-2)$

• Check your answers. Do you need more practice? If so, do Batch H.

Batch H Solve these equations. Show all your working out.

1. $7(n+2) = 2n+19$ 2. $6(a-5) = 5(a-4)$ 3. $8(x+1) = 2(x+16)$
 4. $3(m+1) = m+11$ 5. $9(k-2) = 5(k+2)$ 6. $8m-26 = 3(3+m)$
 7. $9(e-2) = 8(e-1)$ 8. $4(t-6) = 3(t-1)$ 9. $3b+4(b+3)+2=0$

• Check your answers.

Star Challenge

All correct = 1 star

Find the fours

$z-2 = \frac{1}{2}z+1$	$12-2x=2$	$3r-4=2$	$\frac{x+3}{3}=3$	$7c-4=3$	$\sqrt{n}=2$	$x^2-5=20$	$\frac{y}{2}=1$
$m-2=1$	$\frac{2(3e+1)}{14}$	$\frac{6}{m+1}=3$	$3(6-t)=3$	$\frac{n+6}{2}=5$	$x^2-x=30$	$\frac{3+c}{2c+1}$	$\frac{d+5}{2}=3$
$(n+1)^2=4$	$15-2p=3$	$7-2r=3$	$\frac{4}{a}+4=5$	$\frac{1}{2}(x-4)=1$	$x^2-7=2$	$\frac{6}{c}=2$	$p-3=4$
$v^2=25$	$\frac{1}{3}x+2=3$	$3b-3=9$	$\frac{x+5}{2}=3$	$\frac{1}{2}(3s-1)=4$	$4z-5=15$	$2n-1=1$	$5c+2=14$
$\frac{q+4}{2}=5$	$\frac{1}{2}x^2=8$	$3^n=9$	$\frac{45-40k}{5}=5$	$\frac{1}{2}(10+u)=2$	$2m-1=7$	$\frac{1}{2}d+2=3$	$a+4=9$
$10-3p=1$	$\frac{12}{a}-1=2$	$17-3m=5$	$\frac{8}{k-2}=4$	$n^2=4n$	$12-q=8$	$\frac{48}{t}=12$	$3(t+1)=9$
$5p-6=4$	$\frac{3+5t}{4(t+1)}$	$5y+10=5$	$\frac{25+y}{5}=6$	$3b-8=10$	$\frac{11+a}{3}=5$	$\frac{10}{x-1}=5$	$3v-1=2v$
$\frac{24}{t+1}=3$	$17-3m=2$	$7-h=1$	$2p+1=3p$	$\frac{15}{w}=5$	$3p-5=7$	$12-5p=2$	$5(t-5)=5$

Fourteen of these equations have a solution equal to 4.
 Find which equations they are.
 Shade in the squares for these equations.

• Your teacher has the answer to this.

Equation crossnumber

Across

1. $\frac{n}{4} = 30$
3. $2m - 1 = 25$
5. $3p - 2 = 40$
6. $100 - 3t = 10$
8. $2(x + 1) = 46$
9. $v + 1 = 125$
10. $2w - 7 = w + 8$
11. $2c - 62 = c + 69$
14. $4(3e - 49) = 80$
16. $\sqrt{x} = 25$
18. $7(n + 1) = 5n + 13$
19. $3(e - 4) = 2e + 2$
20. $5(n - 3) = 50$
22. $4(t + 1) = 2(t + 11)$
23. $\sqrt{k} = 10$

1	2			3			4
			5			6	
7		8			9		
10			11	12			
		13				14	15
	16		17		18		
19			20			21	
		22			23		

Down

2. $t - 3 = 18$
3. $5x + 5 = 75$
4. $\sqrt{(m - 4)} = 10$
5. $\frac{t}{11} = 11$
6. $40 - p = 8$
7. $\frac{v}{7} = 30$
9. $35 - 3p = 2$
12. $n - 2 = 30$
13. $\frac{1}{4}x = 18$
15. $t - 1 = 30$
16. $\sqrt{x} = 8$
17. $2(n - 1) = 100$
19. $3t - 1 = 50$
21. $2m = 100$

Take a double spread in your book.

Stick this worksheet on the left hand page.

Show all working out on the right hand page.

• Your teacher has the answers to these.

Section 8: Simultaneous equations

Level
7

In this section you will learn how to solve simultaneous equations algebraically.

DEVELOPMENT

D8.1: The elimination method

$3n + 4 = 7$ is an equation with one unknown.

To solve it you must find the value of the unknown that satisfies the equation.

$$3n + 4m = 27$$

$2n - m = 7$ are a pair of **simultaneous equations**.

There are two unknowns. To solve these equations you must find the values of both unknowns that satisfy the two equations.

EXAMPLE

Q: Ron buys 3 cans of drink and 2 packets of crisps. He spends 90p
Erin buys 5 cans of drink and 2 packets of crisps. She spends £1.30

How much do the drinks and crisps each cost ?

A: $3D + 2C = 90$

$5D + 2C = 130$

	$5D + 2C = 130$ — (1)	
First write the equations out one below the other.	$3D + 2C = 90$ — (2)	Subtract one equation from the other.
	$2D = 40$	
	$D = 20$	so drinks cost 20p

Putting $D = 20$ into (1) $100 + 2C = 130$

$$2C = 30$$

$$C = 15$$

so crisps cost 15p

For each problem below:

- write out the equations one below the other;
- solve the equations, showing all working;
- state the value of each of the unknowns.

1. $4p + 3q = 17$ 2. $2a + 2b = 23$
 $5p - 3q = 1$ $2a + b = 13$

3. $2m + 3n = 15$ 4. $5x + y = 17$
 $m + 3n = 0$

5. Look at the working out for each of questions 1 – 4.
 (a) How can you tell which unknown (letter) will disappear when you + or – ?
 (b) How do you know whether to + or – if you are not told ?

Solve the pairs of equations below in exactly the same way as you did in questions 1 – 4. But, this time, YOU must decide whether to + or –.

6. $3s + 5t = 25$ 7. $4e - f = 14$ 8. $10g + 3h = 4$
 $3s - t = 13$ $3e + f = 7$ $7g + 3h = 1$

- Check your answers.

D8.2: When you can't just add or subtract

EXAMPLE Solve $3x + 2y = 19$
 $2x + y = 12$

There's a difficulty here.

Neither the x's nor the y's have the same numbers in front.

But - what if I multiply the second equation by 2?

I get

$$\begin{array}{rcl} 3x + 2y & = & 19 \quad \text{--- (1)} \\ 4x + 2y & = & 24 \quad \text{--- (2)} \\ x & = & 5 \quad \text{Subtracting (1) from (2)} \end{array}$$

Putting $x = 5$ into (1) $15 + 2y = 19$
 $2y = 4$
 $y = 2$ $x = 5$ & $y = 2$

For each problem below:

- write out the equations one below the other
- solve the equations, showing all working
- state the value of each of the unknowns

- | | | |
|-----------------------------------|-------------------------------------|-------------------------------------|
| 1. $2y + 2z = 8$
$3y + z = 14$ | 2. $3h + 4j = 10$
$h + 2j = 6$ | 3. $5c + 4d = 17$
$3c - 2d = -3$ |
| 4. $2p - 3q = 8$
$p + q = 9$ | 5. $2m + 5n = 26$
$-m + 4n = 13$ | 6. $5u + 2v = 34$
$u + v = 8$ |
| 7. $2s - t = 0$
$3s + 2t = 21$ | 8. $3s + 4t = -5$
$s - 2t = -5$ | |

- Check your answers.

D8.3: You're on your own

In the last set of problems, Ruff thought his way round the difficulty for you.
 This time YOU've got to do the thinking.

For each problem below:

- write out the equations one below the other;
- solve the equations, showing all working;
- state the value of each of the unknowns.

- | | | |
|-------------------------------------|---------------------------------------|---------------------------------------|
| 1. $3x - 2y = 17$
$2x + 3y = 20$ | 2. $6m - 2n = 2$
$4m + 3n = 23$ | 3. $2v + 3w = 0$
$7v + 2w = 17$ |
| 4. $5s + 3t = 33$
$2s - 2t = 10$ | 5. $2p - 5q = 13$
$-3p + 2q = -14$ | 6. $11m - 13n = 1$
$2m - 5n = -13$ |

- Check your answers.

D8.4: The substitution method

EXAMPLE Q: Solve $y = 2x$
 $3x + 5y = 39$

Step 1: Write down the equations.

$$y = 2x \quad \text{--- (1)}$$
$$3x + 5y = 39 \quad \text{--- (2)}$$

Step 2: $3x + 5(2x) = 39$ Replace y in (2) by $2x$ since $y = 2x$

Step 3: $13x = 39$
 $x = 3$

Step 4: $y = 6$ Put x into (1) to get y

Solve by substitution, showing all working:

1. $u = 3v$ $2u + 3v = 27$	2. $p = 2q$ $5p - 7q = 12$	3. $y = 2x + 1$ $3y - 2x = 11$
4. $s = 10 - 2t$ $2s + 3t = 18$	5. $p = 7 - q$ $5p - 2q = 7$	6. $c = 3 + 2d$ $3c + 5d = 11$

• Check your answers.

D8.5: Rearrange and substitute

EXAMPLE Q: Solve $y - x = 3$ & $2x + 3y = 34$

Step 1: Rearrange $y - x = 3$ to get $y = x + 3$

$$y = x + 3 \quad \text{--- (1)}$$
$$2x + 3y = 34 \quad \text{--- (2)}$$

Then solve (1) and (2) as in D8.4

Rearrange and solve by substitution. Show all working:

1. $m - n = 1$ $4m - 3n = 8$	2. $a = 2b + 3$ $4b - a = 1$	3. $3p + q = 5$ $2q - p = 3$
---------------------------------	---------------------------------	---------------------------------

• Check your answers.

PRACTICE

P8.6: Choose your own method

Solve each of these equations. Use any method you choose. Show all working.

1. $u + 2v = 6$ $3u - 2v = 10$	2. $y = 3x - 1$ $2y + x = 12$	3. $x + y = 0$ $2x + 5y = 3$
4. $p - q = 0$ $4p + 5q = 27$	5. $7w - 2x = 9$ $4w + x = 18$	6. $4m - 3n = 18$ $-m - 4n = 5$
7. $5s + 2t = 24$ $4s - 3t = -13$	8. $5x + 2y = 16$ $7x - 4y = 19$	9. $e - f = 4$ $5f - e = 8$

• Check your answers.

Section 9: Multiplying out two brackets

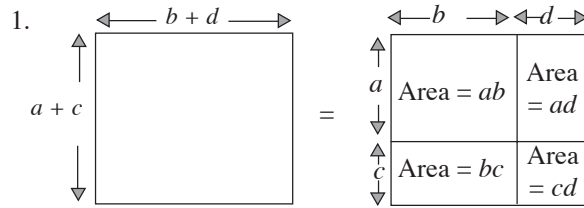
In this section you will:

- learn to multiply out two brackets
- look at why this method works

Level
8

DEVELOPMENT

D9.1: Multiplying two brackets together

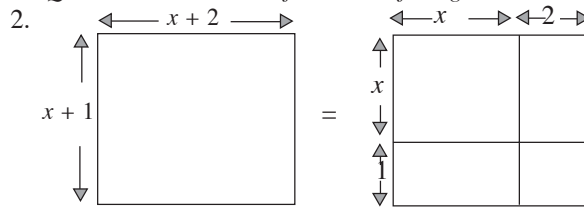


Percy Nunn, who devised this method of showing how to multiply out brackets, was so pleased with it that he put it on the front cover of his textbook, published in 1913.

Complete this statement:

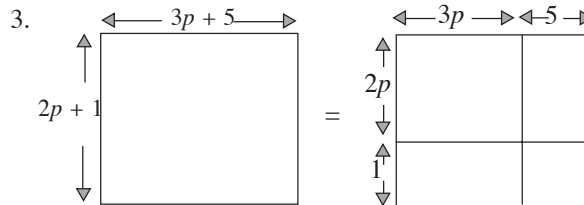
$$(a+c)(b+d) = ac + \dots + \dots + \dots$$

In Q 2-3, what do each of the sets of diagrams show?



Note:

$$\begin{aligned} 2p \times 3p \\ = 2 \times p \times 3 \times p \\ = 6p^2 \end{aligned}$$



Hukka

4. Multiply out $(a+2)(b+3)$

5. Multiply out $(x+y)(x+z)$

• Check your answers.

PRACTICE

P9.2: Practice in multiplying two brackets together

Draw diagrams and multiply out the brackets:

- $(a+1)(b+2)$
- $(m+n)(2m-n)$
- $(3y+2)(y-2)$
- $(c+1)(c+5)$
- $(p+q)(p+2q)$
- $(2p+1)(2p+3)$
- $(p+4)(p-1)$
- $(5+c)(4-c)$
- $(c-5)(c+2)$

• Check your answers.

D9.3: A shorter method for multiplying two brackets

<p>EXAMPLE Q: Multiply out $(2x + 3)(x - 5)$</p> <p>A: $(2x + 3)(x - 5) = 2x(x - 5) + 3(x - 5)$ $= 2x^2 - 10x + 3x - 15$ $= 2x^2 - 7x - 15$</p>	<p>After some practice, you should be able to omit this line.</p>
---	---

Multiply out:

- | | | |
|-----------------------|------------------------|-----------------------|
| 1. $(2a - 1)(a + 2)$ | 2. $(g + h)(2g + h)$ | 3. $(2p - 9)(3p + 1)$ |
| 4. $(4 + m)(5 - m)$ | 5. $(b - 1)(2b + 5)$ | 6. $(5u - 2)(4u + 3)$ |
| 7. $(x - y)(x + 2y)$ | 8. $(n + 10)(n - 3)$ | 9. $(d + 5)(d - 7)$ |
| 10. $(e - 1)(2e + 3)$ | 11. $(2v - 3)(3v - 2)$ | 12. $(x + 3)(x + 3)$ |
| 13. $(b + 2)(b - 3)$ | 14. $(2x + 1)(x + 1)$ | 15. $(r + 3)(2r - 3)$ |

• Check your answers.

D9.4: Squaring brackets

<p>EXAMPLE Q: Expand $(x - 3)^2$</p> <p>A: $(x - 3)^2 = (x - 3)(x - 3)$ $= x^2 - 6x + 9$</p>
--

- Expand: 1. $(a + 2)^2$ 2. $(2x - 1)^2$ 3. $(3p + 5)^2$ 4. $(a + b)^2$

• Check your answers.

D9.5: A shortcut you should learn

<p>You need to be able to write down, without any working, the expansion of any expression like $(a + b)^2$</p> <p>$(a + b)^2 = a^2 + b^2 + 2ab$</p> <p>the sum of the squares plus twice the product</p>

Write down the expansions of:

- | | | | |
|-----------------|-----------------|-----------------|-----------------|
| 1. $(b + 5)^2$ | 2. $(x + 6)^2$ | 3. $(2v + 1)^2$ | 4. $(3t - 4)^2$ |
| 5. $(4c - 3)^2$ | 6. $(a + 2b)^2$ | 7. $(5p + q)^2$ | 8. $(7t - 3)^2$ |

• Check your answers.

<p>Star Challenge 13H-13H</p> <p>Expand and simplify:</p> <table border="0"> <tr> <td>1. $(3p + 1)(p + 2)$</td> <td>2. $(a + 5)(2a - 5)$</td> <td>3. $(6u + v)(2u - v)$</td> </tr> <tr> <td>4. $(2n + 11)(3 - n)$</td> <td>5. $(x - 1)(x + 1)$</td> <td>6. $(3t - 2)(2t + 3)$</td> </tr> <tr> <td>7. $(x - y)^2$</td> <td>8. $(n + 9)^2$</td> <td>9. $(4p + 5)^2$</td> </tr> </table> <p>Without any working out, write down the expansions of:</p> <table border="0"> <tr> <td>10. $(x + 2)(x - 3)$</td> <td>11. $(3m - 2)(m + 1)$</td> <td>12. $(5e - 1)(3e + 1)$</td> </tr> <tr> <td>13. $(x - 5)^2$</td> <td>14. $(r + 7)^2$</td> <td>15. $(2v - t)^2$</td> </tr> </table>	1. $(3p + 1)(p + 2)$	2. $(a + 5)(2a - 5)$	3. $(6u + v)(2u - v)$	4. $(2n + 11)(3 - n)$	5. $(x - 1)(x + 1)$	6. $(3t - 2)(2t + 3)$	7. $(x - y)^2$	8. $(n + 9)^2$	9. $(4p + 5)^2$	10. $(x + 2)(x - 3)$	11. $(3m - 2)(m + 1)$	12. $(5e - 1)(3e + 1)$	13. $(x - 5)^2$	14. $(r + 7)^2$	15. $(2v - t)^2$	<p>15 correct = 2 stars 13-14 correct = 1 star</p>
1. $(3p + 1)(p + 2)$	2. $(a + 5)(2a - 5)$	3. $(6u + v)(2u - v)$														
4. $(2n + 11)(3 - n)$	5. $(x - 1)(x + 1)$	6. $(3t - 2)(2t + 3)$														
7. $(x - y)^2$	8. $(n + 9)^2$	9. $(4p + 5)^2$														
10. $(x + 2)(x - 3)$	11. $(3m - 2)(m + 1)$	12. $(5e - 1)(3e + 1)$														
13. $(x - 5)^2$	14. $(r + 7)^2$	15. $(2v - t)^2$														

Patterns and Rules

Section 1: Rules in words p208

D1.1: Rules for number chains

- 4 → 7 → 10 → 13 → 16
- 13 → 11 → 9 → 7 → 5
- 2 → 4 → 8 → 16 → 32
- 3 → 7 → 15 → 31 → 63
- 4 → 10 → 22 → 46 → 94
- 32 → 16 → 8 → 4 → 2
- 20 → 12 → 8 → 6 → 5
- 4 → 6 → 10 → 18 → 34
- Add 3
- Take away 10
- Multiply by 2
- Divide by 2 or halve it
- Add 4 more than last time

D1.2: From tables to rules

- Number of sweets = 15 x number of packets
- Postage = 20 x number of letters
- Perimeter = 4 x number of diamonds
- Number of dots = number of lines + 1

D1.3: Number machines

Rule in words	Number machine	Table
1.		In 5 10 12 Out 9 14 16
2.	- 3	In 4 8 10 Out 1 5 7
3.	x 3	In 2 5 7 Out 6 15 21
	Rule in words	Number machine
4. Multiply by 5		In 2 4 6 Out 10 20 30
5.		In 2 5 7 Out 4 25 49
6.		In 2 4 5 Out 5 9 11
7.		In 2 4 5 Out 6 10 12
8. No		

Section 2: Using letters for numbers p210

D2.1: Letters for unknown numbers

- $n + 2$
- $n - 2$
- $2n$ & $n + n$
- $2n + 1$
- $3n$
- $2n + 3$
- $3n + 1$
- A is $s + 2$ B is $2s$ C is $s - 2$ D is $2s + 2$

D2.2: Expressions with brackets

- No
- Yes
- Number of sweets = $2 \times (n + 3) = 2(n + 3)$
and $2(n + 3) = 2n + 6$
- (a) $n - 3$ (b) $2(n - 3)$

D2.3: Matching pairs of algebraic cards

Task 1: $n + n = 2n$ $n^2 = n \times nn + n + 1 = 2n + 1$

Task 2: $n + n + n = 2n + n$ $2n + 1 = 1 + 2n$
 $2 + n + 3 = n + 5$

Section 3: Machines and rules p212

D3.1: Rules for machines

- (a) 14 (b) 11 (c) $3N + 2$ (d) $N \rightarrow 3N + 2$
- $N \rightarrow 4N - 1$ 3. $N \rightarrow N^2 + 2$
- $N \rightarrow 5N + 1$ 5. $N \rightarrow 5N - 3$
- $N \rightarrow 2(N + 1)$
- (a) $x3 + 1$ (b) $+1 \times 3$ 8. x itself $+2$

D3.2: Machine instructions

- Machines 1 & 6 Machines 2 & 4
Machines 3 & 5
- Machines 1 & 6 Machines 2 & 4
Machines 3 & 5

D3.3: Letters for In and Out

- Out is 7 2 4
- $d = c + 5$
- Out is 1 3 5
- $q = 3p + 2$

Section 4: Algebra rules, OK ? p214

P4.1: Table problems

1. <table border="1"><tr><td>7</td><td>9</td></tr><tr><td>5</td><td>7</td></tr></table>	7	9	5	7	2. <table border="1"><tr><td>13</td><td>5</td></tr><tr><td>14</td><td>6</td></tr></table>	13	5	14	6	3. <table border="1"><tr><td>22</td><td>12</td></tr><tr><td>14</td><td>4</td></tr></table>	22	12	14	4	4. <table border="1"><tr><td>14</td><td>16</td></tr><tr><td>6</td><td>8</td></tr></table>	14	16	6	8
7	9																		
5	7																		
13	5																		
14	6																		
22	12																		
14	4																		
14	16																		
6	8																		
5. <table border="1"><tr><td>2</td><td>4</td></tr><tr><td>6</td><td>8</td></tr></table>	2	4	6	8	6. <table border="1"><tr><td>-1</td><td>3</td></tr><tr><td>1</td><td>5</td></tr></table>	-1	3	1	5	7. <table border="1"><tr><td>2</td><td>5</td></tr><tr><td>3</td><td>6</td></tr></table>	2	5	3	6	8. <table border="1"><tr><td>2</td><td>3</td></tr><tr><td>4</td><td>6</td></tr></table>	2	3	4	6
2	4																		
6	8																		
-1	3																		
1	5																		
2	5																		
3	6																		
2	3																		
4	6																		
9. <table border="1"><tr><td>2</td><td>0.5</td></tr><tr><td>-4</td><td>-1</td></tr></table>	2	0.5	-4	-1	10. <table border="1"><tr><td>7</td><td>-5</td></tr><tr><td>9</td><td>-7</td></tr></table>	7	-5	9	-7	11. <table border="1"><tr><td>7</td><td>12</td></tr><tr><td>2</td><td>7</td></tr></table>	7	12	2	7	12. <table border="1"><tr><td>15</td><td>11</td></tr><tr><td>22</td><td>18</td></tr></table>	15	11	22	18
2	0.5																		
-4	-1																		
7	-5																		
9	-7																		
7	12																		
2	7																		
15	11																		
22	18																		

P4.2: Number chains

- 4, 8, 14, 2, 4, 10
- 9, 6, 6, 4, 9, 5
- 7, 4, 1, 2, 12, 3
- 5, 2, 4, 6, 4, 8
- 5
- 1, -3, 0, 1, 2
- 2, -3, 8, 3, -3, 4
- 3, 2, 1, 4, 32, 16
- 2, 12, 2, 36, 4, 1

Section 5: Shape patterns p220

D5.1: Rules for shape numbers

- (L) 1 2 3 (D) 2 3 4
- (a) 21 (b) $D = L + 1$
- (L) 1 2 3 (D) 4 8 12
- Number of matches = $4 \times$ shape number
- $M = 4S$
- 40

D5.2: Choose the right formula

- $S = 2L$
- $P = 2S + 2$
- $M = 3S + 1$
- $d = 3p + 2$
- 12
- 32

D5.3: Matchstick patterns

- (a) 21 (b) 26 (c) Add 5 (d) 51
- Set A:

1	2	3	4	5	10
3	5	7	9	11	21

 Add 2
- Set B:

1	2	3	4	5	10
4	7	10	13	16	31

 Add 3
- Set C:

1	2	3	4	5	6
10	17	24	31	38	45

 Add 7
- Set D:

1	2	3	4
4	10	18	28

 Add 2 more than last time
- Set E:

1	2	3	4
9	18	30	45

 Add 3 more than last time

E5.4: Dotty number patterns

- P crossnumbers 1 5 9 13 17 21 25
Q square numbers 1 4 9 16 25 36 49
R triangle numbers 1 3 6 10 15 21 28
S diamond numbers 1 5 13 25 41 61 85

Section 6: Solving equations p224

D6.1: What is an equation ?

- 5
- 10
- 6
- 3
- $N = 7$
- $N = 4$
- $N = 10$
- $N = 3$
- $N = 5$
- $N = 4$
- $N = 10$
- $N = 7$

Section 7: Systematic eq. solving p225**D7.1: Diagnostic Testing****Diagnostic Test for D7.2**

$$\begin{array}{lll} 1. 3p + 2 = 11 & 2. 2c + 40 = 140 & 3. 5t - 3 = 32 \\ 3p = 9 & 2c = 100 & 5t = 35 \\ p = 3 & c = 50 & t = 7 \end{array}$$

Diagnostic Test for D7.4

$$\begin{array}{ll} 1. 3 + 4x = 13 - x & 2. 5m - 6 = 2m + 3 \\ 3 + 5x = 13 & 3m - 6 = 3 \\ 5x = 10 & 3m = 9 \\ x = 2 & m = 3 \\ 3. 6v + 7 = 39 - 2v & \\ 8v + 7 = 39 & \\ 8v = 32 & \\ v = 4 & \end{array}$$

Diagnostic Test for D7.5

$$\begin{array}{ll} 1. 2(3n + 1) = 20 & 2. 3(2p - 5) = 15 \\ 6n + 2 = 20 & 3m - 6 = 3 \\ 6n = 18 & 3m = 9 \\ n = 3 & m = 3 \\ 3. 5(4t + 1) = 65 & \\ 20t + 5 = 65 & \\ 20t = 60 & \\ t = 3 & \end{array}$$

Diagnostic Test for D7.6

$$\begin{array}{ll} 1. 2(p + 1) = p + 8 & 2. 16 + 3a = 2(a + 11) \\ 2p + 2 = p + 8 & 16 + 3a = 2a + 22 \\ p + 2 = 8 & 3a = 2a + 6 \\ p = 6 & a = 6 \\ 3. 3(4e - 6) = 2(3e + 6) & \\ 12e - 18 = 6e + 12 & \\ 6e - 18 = 12 & \\ 6e = 30 & \\ e = 5 & \end{array}$$

D7.2: The basic technique

$$\begin{array}{lll} 1. 5p + 3 = 18 & 2. 3k - 1 = 11 & 3. 4e - 2 = 18 \\ 5p = 15 & 3k = 12 & 4e = 20 \\ p = 3 & k = 4 & e = 5 \\ 4. 3 & 5. 5 & 6. 2 & 7. 3 & 8. 5 & 9. 7 \end{array}$$

P7.3: Basic technique practice**Batch A:**

$$\begin{array}{llll} 1. 5 & 2. 6 & 3. 2 & 4. 3 & 5. 3 \\ 6. 6 & 7. 5 & 8. 3 & 9. 5 & 10. 5 \\ 11. 6 & 12. 5 & 13. 11 & 14. 5 & 15. 9 \end{array}$$

Batch B:

$$\begin{array}{llll} 1. 4 & 2. 11 & 3. 8 & 4. 3 & 5. 8 \\ 6. 10 & 7. 8 & 8. 15 & 9. 5 & 10. 8 \\ 11. 9 & 12. 6 & 13. 5 & 14. 1 & 15. 5 \end{array}$$

D7.4: Equations with letters on both sides**Batch C:**

$$\begin{array}{llll} 1. 5 & 2. 7 & 3. 2 & 4. 7 & 5. 7 \\ 6. 7 & 7. 6 & 8. 9 & 9. 2 & 10. 5 \\ 11. 3 & 12. 2 & & & \end{array}$$

Batch D:

$$\begin{array}{llll} 1. 4 & 2. 3 & 3. 1 & 4. 4 & 5. 3 \\ 6. 4 & 7. 2 & 8. 2 & 9. 10 & 10. 5 \\ 11. 3 & 12. 3 & & & \end{array}$$

D7.5: Simple equations with brackets**Batch E:**

$$\begin{array}{llll} 1. 3 & 2. 3 & 3. 2 & 4. 11 & 5. 3 \\ 6. 37 & 7. 5 & 8. 3 & 9. 11 & \end{array}$$

Batch F:

$$\begin{array}{llll} 1. 4 & 2. 1 & 3. 3 & 4. 5 & 5. 4 \\ 6. 1 & 7. 4 & 8. 7 & 9. 3 & \end{array}$$

D7.6: Letters on both sides and brackets**Batch G:**

$$\begin{array}{llll} 1. 4 & 2. 4 & 3. 10.5 & 4. 3 & 5. 2 \\ 6. 1 & 7. 8 & 8. 2 & 9. 2 & \end{array}$$

Batch H:

$$\begin{array}{llll} 1. 1 & 2. 10 & 3. 4 & 4. 4 & 5. 7 \\ 6. 7 & 7. 10 & 8. 21 & 9. -2 & \end{array}$$

Section 8: Simultaneous equations p230**D8.1: The elimination method**

$$\begin{array}{llll} 1. 2, 3 & 2. 10, 1.5 & 3. 15, -5 & 4. 2, 7 \\ 5. If you do not know, ask your teacher. & & & \\ 6. 5, 2 & 7. 3, -2 & 8. 1, -2 & \end{array}$$

D8.2: When you can't just add or subtract

$$\begin{array}{llll} 1. 5, -1 & 2. -2, 4 & 3. 1, 3 & 4. 7, 2 & 5. 3, 4 \\ 6. 6, 2 & 7. 3, 6 & 8. -3, 1 & \end{array}$$

D8.3: You're on your own

$$1. 7, 2 \quad 2. 2, 5 \quad 3. 3, -2 \quad 4. 6, 1 \quad 5. 4, -1 \quad 6. 6, 5$$

D8.4: The substitution method

$$1. 9, 3 \quad 2. 8, 4 \quad 3. 2, 5 \quad 4. 6, 2 \quad 5. 3, 4 \quad 6. 7, 2$$

D8.5: Rearrange and substitute

$$1. 4, 5 \quad 2. 2, 7 \quad 3. 1, 2$$

P8.6: Choose your own method

$$\begin{array}{llll} 1. 4, 1 & 2. 2, 5 & 3. -1, 1 & 4. 3, 3 & 5. 3, 6 \\ 6. 3, -2 & 7. 2, 7 & 8. 3, 0, 5 & 9. 7, 3 & \end{array}$$

Section 9: Mult. out two brackets p233**D9.1: Multiplying two brackets together**

$$\begin{array}{ll} 1. (a + b)(c + d) = ac + ad + bc + bd \\ 2. (x + 1)(x + 2) = x^2 + 2x + x + 2 = x^2 + 3x + 2 \\ 3. (2p + 1)(3p + 5) = 6p^2 + 10p + 3p + 5 = 6p^2 + 13p + 5 \\ 4. ab + 3a + 2b + 6 & 5. x^2 + xz + xy + yz \end{array}$$

P9.2: Practice in mult. two brackets together

$$\begin{array}{ll} 1. ab + 2a + b + 2 & 2. 2m^2 + mn - n^2 \\ 3. 3y^2 - 4y - 4 & 4. c^2 + 6c + 5 \\ 5. p^2 + 3pq + 2q^2 & 6. 4p^2 + 8p + 3 \\ 7. p^2 + 3p - 4 & 8. 20 - 5c + c^2 & 9. c^2 - 3c - 10 \end{array}$$

D9.3: A shorter method for mult. two brackets

$$\begin{array}{ll} 1. 2a^2 + 3a - 2 & 2. 2g^2 + 3gh + h^2 \\ 3. 6p^2 - 25p - 9 & 4. 20 + m - m^2 \\ 5. 2b^2 + 3b - 5 & 6. 20u^2 + 7u - 6 \\ 7. x^2 + xy + 2y^2 & 8. n^2 + 7n - 30 \\ 9. d^2 - 2d - 35 & 10. 2e^2 + e - 3 \\ 11. 6v^2 - 13v + 6 & 12. x^2 + 6x + 9 \\ 13. b^2 - b - 6 & 14. 2x^2 + 3x + 1 \\ 15. 2r^2 + 3r - 9 \end{array}$$

D9.4: Squaring brackets

$$\begin{array}{ll} 1. a^2 + 4a + 4 & 2. 4x^2 - 4x + 1 \\ 3. 9p^2 + 30p + 25 & 4. a^2 + 2ab + b^2 \end{array}$$

D9.5: A shortcut you should learn

$$\begin{array}{ll} 1. b^2 + 25 + 10a & 2. x^2 + 36 + 12x \\ 3. 4v^2 + 1 + 4v & 4. 9t^2 + 16 - 24t \\ 5. 16c^2 + 9 - 24c & 6. a^2 + 4b^2 + 4ab \\ 7. 25p^2 + q^2 + 10pq & 8. 49t^2 + 9 - 42t \end{array}$$