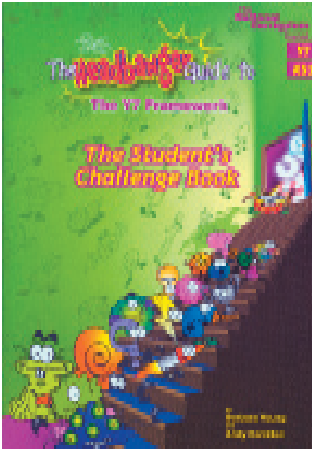


The 'Maths is ...' Jugglers

Knowledge

Skills

Understanding



ISBN-10: 1-874428-97-2

ISBN-13: 978-1-874428-97-8

Sample topic

from the *Headbanger Guide to the* **The Student's Challenge Book**

**You have permission to print off this material
 use it with your students.**

CONTENTS OF THE PACK

How does the Star Challenge system work ?	p2
Star Challenges and High Level Challenges from Topic 10: Number Patterns	p3-12
Teacher's Answers to the Challenges	p13-20

The Student's Challenge Book

- This is intended for use at home.
- Homeworks may be set from it.
- It includes many of the **Star Challenges** from the ends of sections. This extends the use of this text as a homework book to a much wider range of students.
- It also contains ALL of the **200 HIGH LEVEL CHALLENGES**, so students can work on these puzzles and problems whenever they wish.



The **High Level Challenges** were first used in the highly successful predecessor to *Headbanger*, the *Big Edd Guide*.

At Tarporley High School, students in our two top sets loved them. Almost all top set students did every High Level Challenge. Many completed all 152 of them before the end of the year, and came back and asked for more. (So there are now 200 in *Headbanger*.)

But, unexpectedly, many of the students in our two second sets tackled a lot of these problems with great enthusiasm. These students had successes with puzzles and problems that we had thought well beyond them. Confidence and motivation soared and, later, so did exam results.

Other schools using this material report similar reactions.

**Order half price copies of Y7 Headbanger and/or The Student's Challenge Book
 using the Special Offer form
 which can be downloaded from the website.**

www.mathsisjugglers.co.uk

How does the Star Challenge system work ?

The number of stars is a measure of the difficulty of the Challenge.

So, one star denotes a basic fairly easy challenge.

However, students should not be discouraged from attempting any Challenges.

For example, students may earn fewer stars from many two or three star challenges, with partially correct answers.

Students of lower ability can amaze you with what they can achieve, if they really want to tackle problems that you might think beyond them.

A true cautionary tale

Three very low ability students were working together. They had to. Mark was severely dyslexic - he could not read at all. Jason could read very well, but could make no sense of what he was reading. Matthew was very slow at everything. Between them, they could tackle problems.

Jason would read the question. Mark would explain what it was all about. Together the three of them would work out how to solve the problem.

One day they had done all the one star problems. They asked me if they could try the three star problem - The Chest of Drawers. It contained a mixture of fractions and decimals and both cm and mm. My first reaction was to tell them that it was too difficult for them - but I said they could try it, provided they accepted that they might not be able to do it.

About ten minutes later, they brought me the correct answer. I couldn't believe that they had solved such a complex problem. I changed the data (marked below in blue). They were back a few minutes later with the correct answer. Then I made a big mistake. I asked them to tell me how they had worked it out. I couldn't follow their explanation at all !!

From then on I never restricted the choice of Star Challenges of even the least able students.

Star Challenge

The chest of drawers

A chest of drawers 84 cm high has a 42.5 mm top and a 90 mm base. Four identical drawers fit in between, with 22.5 mm between each drawer.

Work out the height of each drawer in cm.

If the 22.5 mm gaps were reduced to 12.5 mm each, what would be the height of each drawer then?

84 cm

$4\frac{1}{4}$ cm

$2\frac{1}{4}$ cm

9 cm

What do schools do with the stars students earn for Star Challenges ?

One school:

- gives a house point for each star.
- has a commendation system and awards a commendation for so many stars. They also award stars for effort when the books are taken in each fortnight.
- gives a smiley face for 5 stars
- gives a Mars Bar for 10 stars
- gives a copy of one of a group of Chaos posters for 20 stars (very popular with low attainers)
-

It doesn't matter what you do. It is the sense of achievement that is most important.

However, a system that awards something for so many stars, then starts them collecting the next set of stars, means that students keep track of how many stars they have and how many more they need towards the next 'prize' – instead of (possibly invidious) comparisons of the total number of stars each student has.

Star Challenge **1**

Topic 10: Number Patterns

All correct = 1 star

16380 is divisible by ... ?

Numbers to try 2 3 4 5 6 7
 8 9 10 11 12 13



Big Edd



16380 is divisible by

16380 is not divisible by

Try each of these numbers and put them into the correct sets.

• Your teacher has the answers to these.

Star Challenge **2 2 2**

Using divisibility rules



45 marks = 3 stars
 40-41 marks = 2 stars
 37-40 marks = 1 star

Task 1: There are 11 numbers here that are divisible by 5. Find them. Put them in ascending order.

Task 2: There are 14 numbers here that are divisible by 2. Find them. Put them in descending order.

Task 3: There are 12 numbers here that are divisible by 3. Find them. Put them in ascending order.

24	47	29	85	70	38
73	95	22	452	711	
215	666	999	4545	8432	631
123	252	170	290	205	
275	746	649	570	891	639
3390	1437	2389	542	797	

Task 4: There are 4 numbers here that are divisible by 4. Find them. Put them in descending order.



Frizzbang

1 mark for each correct number +
 1 mark for each correct order

• Your teacher has the answers to these.

Star Challenge



2 stars for all 3-digit palindromic multiples of 7
2 stars for all 4-digit palindromic multiples of 7

Palindromic multiples of 7

1. $7 \times 1 = 7$ $7 \times 2 = 14$ $7 \times 3 = 21 \dots$

Copy out these multiples of 7 and continue them up to 7×10 .

$7 \times \square\square = 3\square4$



What must this digit be? [Look at the multiples you have already written down.]

Now try multiplying 7 by numbers ending in this digit to complete the statement.

2. $7 \times \square\square = 1\square1$

Use the same technique to work out what this statement could be.

This is a palindromic multiple of 7, because the multiple reads the same backwards as it does forwards.

3. Find the next four palindromic multiples of 7 and write them out in the form

$7 \times \dots = \dots$

If you can see a pattern, describe it.

4. If you can see the pattern, use it to find all the 3-digit palindromic multiples of 7. If you cannot see the pattern, use the original method to find them.

5. Find two 4-digit palindromic multiples of 7 that start and end with a 1.

6. There are eighteen 4-digit palindromic multiples of 7. Find them.

Star Challenge



1 star for each correct pattern and explanation – up to a maximum of 3 stars.

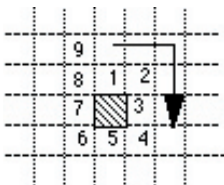
Spiral multiples of 4

Shade one small square in the middle of a sheet of squared paper.



Write the numbers 1,2,3,4,5.....

in a spiral which starts :



As you draw the spiral, shade in the multiples of 4. Explain the pattern that they make.

Possible extensions :

What different patterns do you get if you vary the shape and/or size of the “shaded island” that you put in the centre of your spiral ?

1 star for each correct pattern and explanation – up to a maximum of 3 stars.

Star Challenge 4•4

8 correct = 2 stars
6-7 correct = 1 star

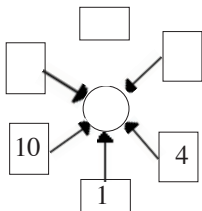
Find all the factors of each number :

- | | | | |
|-------|-------|-------|-------|
| 1. 9 | 2. 10 | 3. 16 | 4. 27 |
| 5. 60 | 6. 90 | 7. 51 | 8. 84 |

• Your teacher has the answers to these.

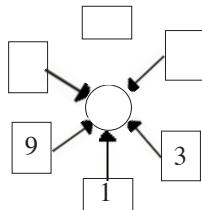
Star Challenge 5•5

16 marks = 2 stars
11-15 marks = 1 star

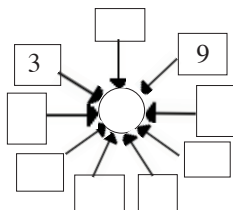


Factor diagram 1
(4 marks)

Copy and complete
each factor
diagram



Factor diagram 2
(4 marks)



Factor diagram 3
(8 marks)



Fission



Chyps

• Your teacher has the answers to these.

Star Challenge 6•6•6

1 star for each correct
number and its 12 factors

$60 = 1 \times 60$	$60 = 4 \times 15$
$60 = 2 \times 30$	$60 = 5 \times 12$
$60 = 3 \times 20$	$60 = 6 \times 10$

60 has 12 factors

Find three more numbers, less than 100, that have EXACTLY 12 factors.
Show all your working.

• Your teacher has the answers to these.

Star Challenge 8 8 8

Emirps

An **emirp** is a prime number which, when reversed, is also prime.

31 and 13 is one set of emirps.

How many more can you find ?

You cannot use single digit prime numbers.

• *Your teacher has the answers to these.*

TARGETS:	3 good	(1 star)
	5 very good	(2 stars)
	more than 5 superb!	(3 stars)

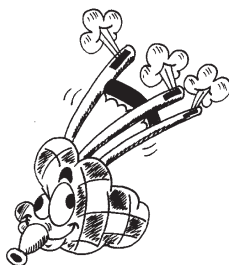
Star Challenge 9 9 9

2-digit primes

1. A 2-digit prime number cannot end in 2. Why ?
2. What other digits can it *not* end in ?
3. There are twenty one 2-digit prime numbers.

Complete this table of 2-digit primes.

11	13	17	19
	23		
		37	
	53		
			89



Baggy

• *Your teacher has the answers to these.*

21 2-digit primes = 3 stars
19-20 2-digit primes = 2 stars
16-18 2 digit primes = 1 star

Star Challenge 10 10

Sum-square primes

8-9 correct = 2 stars
6-7 correct = 1 star

13 is a 2-digit prime number

13 can be written as the sum of two square numbers $13 = 4 + 9$

There are ten 2-digit prime numbers that can be written as the sum of two square numbers.

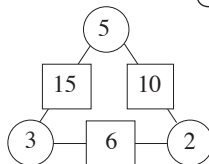
Find the other nine 2-digit prime numbers.

Show all working.

• *Your teacher has the answers to these.*

The number in each is the product of the two numbers on either side of it.

Copy each arithmogon and fill in the missing numbers.



Big Edd

- 1.
- 2.
- 3.
- 4.
- 5.
- 6.
- 7.
- 8.
- 9.

• Your teacher has the answers to these.

Simplification can be done in several stages – BUT, the fraction is simplified in one step if you divide the numerator and denominator by their HCF,

$$\begin{array}{c} \div 6 \\ \frac{30}{48} = \frac{5}{8} \\ \div 6 \end{array}$$

Dividing the numerator and denominator of a fraction by the same number gives an equivalent fraction.

Reduce each fraction to its simplest form *in one stage*.
Show how you do it.

1. $\frac{18}{24}$ 2. $\frac{30}{45}$ 3. $\frac{90}{120}$ 4. $\frac{60}{72}$ 5. $\frac{90}{100}$ 6. $\frac{60}{100}$
7. $\frac{16}{24}$ 8. $\frac{25}{45}$ 9. $\frac{17}{51}$ 10. $\frac{45}{60}$ 11. $\frac{36}{72}$ 12. $\frac{50}{125}$

• Your teacher has the answers to these.

Topic 10: High Level Challenges

EXTENSIONS

YOUR TEACHER HAS THE ANSWERS TO THESE.

Ch 1: Palindromic dates and times

51 correct = 2 stars
48-40 correct = 1 star

A palindrome reads the same backwards as forwards.

RADAR is a palindromic word

12321 and 4774 are palindromic numbers

1. 2002 is a palindromic year.

What was the last palindromic year before 2002 ?

2. 04.40 is a palindromic time on a digital clock. There are 18 palindromic times on a 24 hour digital clock. How many can you find ?

3. 18/4/81 and 31/8/83 are palindromic dates. There are 32 palindromic dates between 1/1/90 and 31/12/92. How many can you find ?

Ch 2: Square palindromes

15-16 marks = 2 stars
12-14 marks = 1 star

Task 1: Using a calculator, if necessary, copy and complete:

$$1 \times 1 =$$

$$11 \times 11 =$$

$$111 \times 111 =$$

$$1111 \times 1111 =$$

(4 marks)

Look for a pattern. Now, without a calculator, copy and complete:

$$11111 \times 11111 =$$

$$111111 \times 111111 =$$

$$1111111 \times 1111111 =$$

$$11111111 \times 11111111 =$$

(8 marks)

Task 2: What is 7×9 ?

What is 77×99 ?

What is 777×999 ? Continue until you have spotted the pattern.

What is 7777777×9999999 ?

(1,1,1,3 marks)

Ch 3: Square headscratchers

3 correct = 2 stars
2 correct = 1 star

1. What is the largest number less than 100 that can be expressed as the sum of two squares ?

2. I am a 3-digit palindromic number.

I am even and square.

When divided by 2, I am not palindromic.

What number am I ?

3. A number, n , is cubed and the answer is squared.

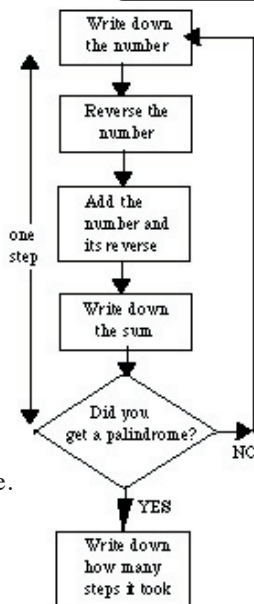
The final number is **46656**. Find n .

Ch 4: Palindromic chains

20 marks = 4 stars
 17-19 marks = 3 stars
 11-16 marks = 2 stars
 9-10 marks = 1 star

- Start with 19. You should get a palindrome after 2 steps.
- How many steps does it take to get a palindrome starting with 12 ?
- How many steps does it take to get a palindrome starting with 651 ?
- Copy and fill in the table below, for all numbers from 11 to 49. (Look for shortcuts !)

Number	Number of steps	Palindrome found
11		
12		
.		
.		
.		
49		



- Look at the numbers that reach 121 in one stage. What are the sums of their digits ? (1 mark)
- What is special about the digits of the numbers that reach 121 in two stages ? (1 mark)
- What are the numbers below 50 that reach 363 in two stages ? Predict another two numbers less than 70 that will reach 363 in two stages and show that they do. Why did you choose those numbers ? (4 marks)
- Which six 2-digit numbers will reach 484 in two stages ? (6 marks)
- Research challenge !** (8 marks)
 Find out what happens to 2-digit numbers with digit sums of 14, 15, 16 & 18.

Ch 5: Digit sums

SECTION 1

All correct = 1 star

- Which of the numbers **57, 93, 218, 369, 205, 924, 7773** is a multiple of 3 ?
- Which of the numbers **47, 63, 198, 279, 405, 714, 8884** is a multiple of 9 ?
- Which of the numbers **27, 36, 237, 372, 809, 673, 376** is a multiple of 6 ?
- Explain how you can tell whether a number is a multiple of 6 without dividing it by 6.

Ch 5: Multiples of what ? (SECTION 2)

All correct = 1 star

Task 1: $74 - 47 = \dots\dots$

$51 - 15 = \dots\dots$

$63 - 36 = \dots\dots$

Try lots like these.
Write down the subtractions and the answers you get.
The answers are always multiples of k.
What is k ?

Task 2: $74 + 47 = \dots\dots$

$51 + 15 = \dots\dots$

$63 + 36 = \dots\dots$

Try lots like these.
Write down the additions and the answers you get.
The answers are always multiples of p.
What is p ?

Ch 6: 37 is three times magic (SECTION 2)

All correct = 1 star

Complete this table:

37×3	= ...
37×6	= ...
37×9	= ...
$37 \times \dots$	= ...
$37 \times \dots$	= ...

Predict the value of 37×27 .
Write down your prediction.
Check it with a calculator.

What is the connection between the sum of the digits of each answer and its original question ?

Ch 7: Abundant, perfect and deficient numbers

36 marks
= 2 stars
30-35 marks
= 1 star

(SECTION 3)

The **factors** of 8 are 1, 2, 4, 8
The **proper factors** of 8 are 1, 2, 4
The **sum of the proper factors** of 8 is 7

The sum of the proper factors of 8 is less than 8.
So, 8 is said to be a **deficient number**.

If the sum of the proper factors is more than the number,
the number is an **abundant number**.

If the sum of the proper factors equals the number itself,
the number is a **perfect number**.

1. Show that 12 is an abundant number. (2 marks)
2. Show that 15 is a deficient number. (2 marks)
3. Why is 3 a deficient number ? (2 marks)
4. Investigate numbers up to 20 and show whether they are abundant, deficient or perfect. Put your answers in a table like this:

Abundant numbers	
Perfect numbers	
Deficient numbers	

(20 marks)

5. Find the two smallest perfect numbers. (4 marks)
6. Find the first six abundant numbers. (6 marks)

Ch 8: Multiple Headscratcher

SECTION 3

3 correct = 2 stars
2 correct = 1 star

1. N is an unknown number that you have to find from these clues :

72 and 90 are both multiples of N.

N has six factors.

N has exactly three multiples between 80 and 130.

What is the value of N ?

185, 518, 851 all have a factor 37

2. Find three other digits that can be arranged in three different ways to make numbers all with a factor of 37. Give the 3 digits and the 3 numbers.
3. Repeat Q2 with third set of 3 digits.

Ch 9: Sum-square primes

SECTION 4

10 correct = 2 stars
8-9 correct = 1 star

There are twenty one 2-digit prime numbers.

Ten of them can each be expressed as the sum of two square numbers.

How many of these ten can you find ?

Ch 10: Sum-prime squares

SECTION 4

17 marks = 2 stars
15-16 marks = 1 star

1. Express the square numbers from 4 to 144 inclusive as the sum of two prime numbers. [11 marks]
2. What do the sums for the odd square numbers have in common ? [1 mark]
3. Each of the 2-digit even square numbers can be expressed as the sum of two prime numbers in two different ways. Show the two ways for each of these numbers. [5 marks]

Ch 11: Product arithmagon challenge

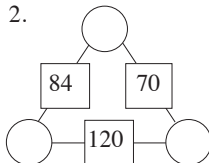
SECTION 5

18 correct = 2 stars
15-17 correct = 1 star

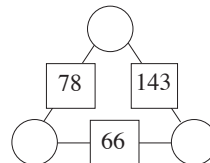
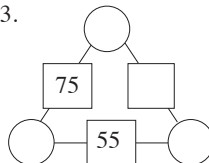
The number in each is the product of the two numbers on either side of it.

Copy each arithmagon and fill in the missing numbers.

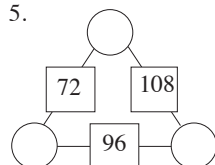
1. 2.



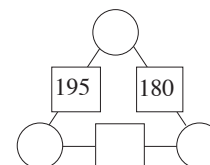
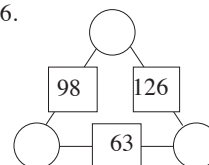
3.



4. 5.



6.



Ch 12: Prime calculator search

SECTION 4



3 correct = 2 stars
2 correct = 1 star

- Two prime numbers multiplied together make **4891**.
What are the numbers ?
- 6887** is also the product of two prime numbers.
What are the numbers ?
- The product of THREE prime numbers is **21199**.
What are the numbers ?



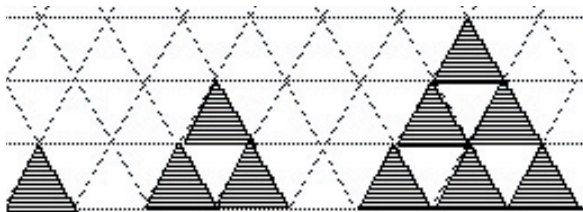
Letmewin

Ch 13: Triangular match patterns

SECTION 6

12 marks = 2 stars
11 marks = 1 star

- These triangles are made with matches :



Δ number 1

Δ number 2

Δ number 3

Continuing the same pattern, draw Δ number 4, Δ number 5 and Δ number 6.

(3 marks)

- Copy and complete this table :

	Δ no 1	Δ no 2	Δ no 3	Δ no 4	Δ no 5	Δ no 6
number of small shaded triangles	1	3				
number of small unshaded triangles		0	1			
total number of small triangles	1	4				
number of matches needed to make shape	3					

(4 marks for all correct, but -1 for each error)

- Explain the pattern in each row of numbers. (4 marks)
- What connection is there between row 1 and row 2 ? (1 mark)

Direct teaching points

Section 1: Divisibility tests

You will need to teach/review the ideas covered by :

D1.1: Dividing whole numbers by 4

D1.2: Testing divisibility by 4

D1.3: Divisibility tests for 3 and 9

You may want to remind them of the divisibility tests for 2,5,10 – but most students can recall them easily. They are also listed at the beginning of **D1.4: Divisibility tests**

A CALCULATOR IS NEEDED FOR **D1.5: DIVISIBILITY USING A CALCULATOR.**

1

All correct = 1 star

16380 is divisible by

16380 is divisible by

2	3	4	5	6
7	9	10	12	13

16380 is not divisible by

8	11
---	----

2

45 marks = 3 stars
40-41 marks = 2 stars
37-39 marks = 1 star

Using divisibility rules

Task 1:	70	85	95	170	205	215	275	290	570	3390	4545			
Task 2:	8432	3390	746	666	570	542	452	290	252	170	70	38	24	22
Task 3:	24	123	252	570	639	666	711	891	999	1437	3390	4545		
Task 4:	8432	452	252	24										

Section 2: Multiples

You will need to teach/review the ideas covered by :

D2.1: “Divisible by” and “multiples of”

D2.2: Lowest common multiples (LCMs)

3

33-34 marks = 2 stars
29-32 marks = 1 star

The Sieve of Eratosthenes

Task 2: Multiples of 4 are also multiples of 2, and so already shaded.		(1 mark)
Task 4: The prime numbers less than 10 are	2 3 5 7	(4 marks)
The prime numbers between 20 and 40 are	23 29 31 37	(4 marks)
The only even prime is	2	(1 mark)
The prime numbers between 70 and 100 are	71 73 79 83 89 97	(6 marks)
Two digit prime numbers always end in	1 or 3 or 7 or 9	(4 marks)
13 is a prime number. Is 31 prime ?	Yes	(1 mark)
41 is a prime number. Is 14 prime ?	No	(1 mark)
First five 3-digit prime numbers	101 103 107 109 113	(5 marks)

Y7 Headbanger: Teachers' Guide

Topic 10

page 82

4 4 4 4

Palindromic multiples of 7

1. $7 \times 52 = 364$
2. $7 \times 23 = 161$
3. $7 \times 36 = 252$
- $7 \times 49 = 343$
- $7 \times 62 = 434$
- $7 \times 75 = 525$

↑
goes up by 13

5&6.	$143 \times 7 = 1001$	$253 \times 7 = 1771$
	$286 \times 7 = 2002$	$396 \times 7 = 2772$
	$429 \times 7 = 3003$	$539 \times 7 = 3773$
	$572 \times 7 = 4004$	$682 \times 7 = 4774$
	5005	5775
	6006	6776
	7007	7777
	8008	8778
	9009	9779

↑
goes up by ?

↑
goes up by ?

2 stars for all 3-digit palindromic multiples of 7
2 stars for all 4-digit palindromic multiples of 7

4. 161, 252, 343, 434, 525, 616, 707, 868, 959

5 5 5

Spiral multiples of 4

For each spiral, check that :

- the numbers follow round the spiral in order
- the multiples of 4 have been shaded.
- the pattern is clearly explained.

1 star for each correct pattern and explanation – up to a maximum of 3 stars

Section 3: Factors

You will need to teach/review the ideas covered by :

D3.1: Guzzintas and factors

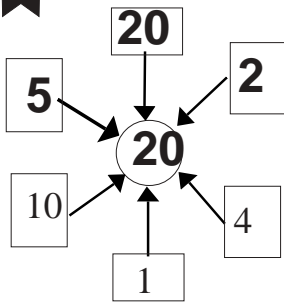
D3.2: Factor pairs

6

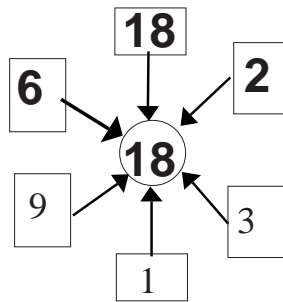
1. 1, 3, 9
2. 1, 2, 5, 10
3. 1, 2, 4, 8, 16
4. 1, 3, 9, 27
5. 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
6. 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90
7. 1, 3, 17, 51
8. 1, 2, 3, 4, 7, 12, 21, 28, 42, 84

8 correct = 2 stars
6-7 correct = 1 star

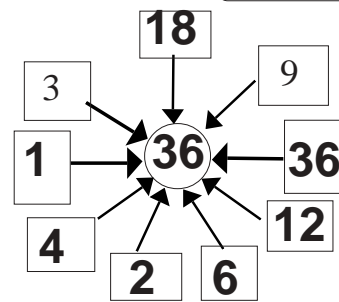
7 7



Factor diagram 1
(4 marks)



Factor diagram 2
(4 marks)



Factor diagram 3
(8 marks)

16 marks = 2 stars
11-15 marks = 1 star

8 8 8

60 has 12 factors

- 72 90 96

1 star for each correct number and its 12 factors

Section 4: Prime numbers

You will need to remind students that a prime number is a number with only two factors. Otherwise, the material is self-explanatory.

9

9-10 marks = 1 star

Primes and squares

1. 23, 29, 31, 37
(4 marks)

2.

1	1	1
4	1, 2, 4	3
9	1, 3, 9	3
16	1, 2, 4, 8, 16	5
25	1, 5, 25	3

(5 marks)

3. an odd number of factors
(1 mark)

Targets

3	good	1 star
5	very good	2 stars
more than 5	superb	3 stars

10 10 10

Emirps

The emirps (primes which are also primes when reversed) below 1000 are given here.

11	13	31	151	347	743	769	967
17	71	157	751	353		787	
37	73	167	761	359	953	797	
79	97	179	971	373		919	
101		181		389	983	929	
107	701	191		709	907		
113	311	199	991	727			
131		313		739	937		
149	941	337	733	757			

21 2-digit primes = 3 stars
19-20 2-digit primes = 2 stars
16-18 2-digit primes = 1 star

11 11 11

2-digit primes

- A 2-digit number ending in 2 has two as a factor.
- 0, 4, 5, 6, 8

3.

11	13	17	19
	23		29
31		37	
41	43	47	
	53		59
61		67	
71	73		79
	83		89
		97	

8-9 correct = 2 stars
6-7 correct = 1 star

12 12

Sum-square primes

$$13 = 9 + 4$$

$$37 = 36 + 1$$

$$61 = 36 + 25$$

$$97 = 81 + 16$$

$$17 = 1 + 16$$

$$41 = 25 + 16$$

$$73 = 64 + 9$$

$$29 = 25 + 4$$

$$53 = 49 + 4$$

$$89 = 64 + 25$$

Section 5: Working with factors

You will need to teach the ideas covered by :

D5.1: Prime factors

D5.2: Highest common factors (HCFs)

13-13

Factor challenge

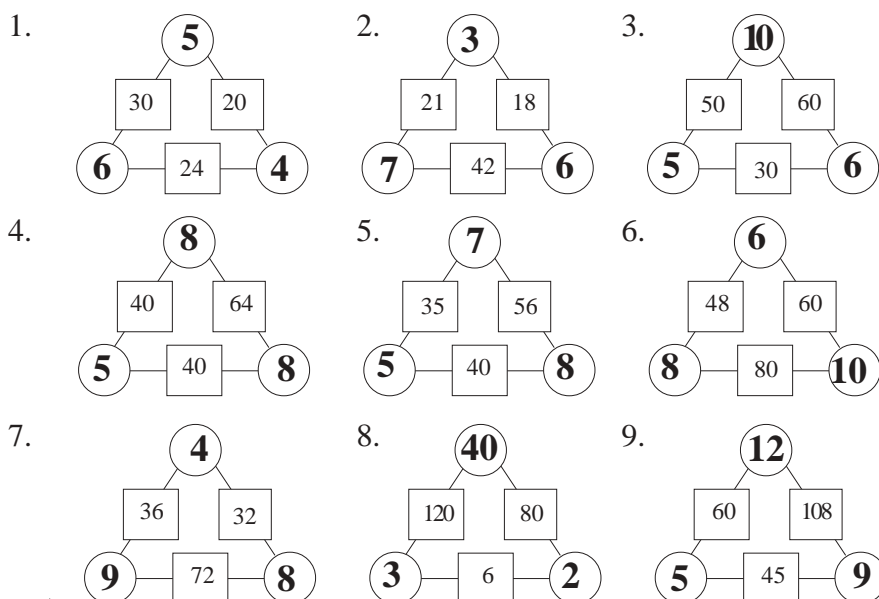
12 factors = 2 stars
10 factors = 1 star

Numbers with 12 factors are 60 72 90 96
Numbers with 10 factors are 48 80

14-14

25-27 correct = 2 stars
19-24 correct = 1 star

Product arithmogons



11-11

11-12 correct = 2 stars
9-10 correct = 1 star

Simplifying fractions in one step

1. $\frac{18}{24} = \frac{3}{4}$ (÷6)

2. $\frac{30}{45} = \frac{2}{3}$ (÷15)

3. $\frac{90}{120} = \frac{3}{4}$ (÷30)

4. $\frac{60}{72} = \frac{5}{6}$ (÷12)

5. $\frac{90}{100} = \frac{9}{10}$ (÷10)

6. $\frac{60}{100} = \frac{3}{5}$ (÷20)

7. $\frac{16}{24} = \frac{2}{3}$ (÷8)

8. $\frac{25}{45} = \frac{5}{9}$ (÷5)

9. $\frac{17}{51} = \frac{1}{3}$ (÷17)

10. $\frac{45}{60} = \frac{3}{4}$ (÷15)

11. $\frac{36}{72} = \frac{1}{2}$ (÷36)

12. $\frac{50}{125} = \frac{2}{5}$ (÷25)

Section 6: Triangle numbers:

D6.1: Investigating triangle numbers needs no introduction.

However, it is essential that each student shows his/her triangle shapes to the teacher. The standard mistake is that the student ignores the directive to make each shape in a different colour. If this is not done, **D6.2: Connecting square and triangle numbers** becomes far more difficult.

16

Triangle patterns

All correct = 1 star

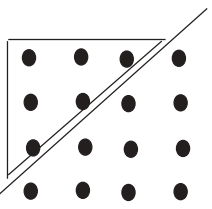
- To make T_8 you add 8 to T_7
- | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|----------|----------|----------|
| T_4 | T_5 | T_6 | T_7 | T_8 | T_8 | T_{10} | T_{11} | T_{12} |
| 10 | 15 | 21 | 28 | 36 | 45 | 55 | 66 | 78 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | |

17

Triangle sum-squares

All correct = 1 star

1. (a)



(b) $m = 3$ $n = 4$

- 36
- 49
- 441

18-18

Triangle sums

12 correct = 2 stars
10-11 correct = 1 star

- $9 = 3 + 6$
- $25 = 10 + 15$
- $18 = 3 + 15$
- $24 = 3 + 21$
- $100 = 45 + 55$
- $51 = 36 + 15$
or $6 + 45$
- $27 = 6 + 21$
- $84 = 6 + 78$
- $87 = 66 + 21$
- $19 = 3 + 6 + 10$
or $1 + 3 + 15$
- $10 = 1 + 3 + 6$
- $31 = 6 + 10 + 15$

19

More shape patterns

All correct = 1 star

Only the numerical parts of the answers are given here.
If they have got any of these wrong, then they have probably got the patterns wrong.

- V-patterns (b) 1, 3, 5, 7, 9, 11, 13, 15, 17, 19
- Hollow square patterns (b) 4, 8, 12, 16, 20, 24, 28, 32, 40
- diamond patterns (b) 1, 4, 9, 16, 25, 36, 49
- hexagonal numbers 1, 7, 19, 37, 61, 91

HIGH LEVEL CHALLENGES FOR TOPIC 10

Ch 1: Palindromic dates and times

49 correct = 2 stars
46-48 correct = 1 star

1. 1991
2. 01.10 02.20 03.30 04.40 05.50 00.00 10.01 11.11 12.21 13.31 14.41 15.51
20.02 21.12 22.22 23.32 ~~24.42~~ ~~25.52~~ [Apologies – there are only 16, not 18]
3. 09/1/90 09/2/90 09/3/90 09/4/90 09/5/90 09/6/90 09/7/90 09/8/90 09/9/90 09/11/90
19/1/91 19/2/91 19/3/91 19/4/91 19/5/91 19/6/91 19/7/91 19/8/91 19/9/91 19/11/91
1/9/91
29/1/92 29/2/92 29/3/92 29/4/92 29/5/92 29/6/92 29/7/92 29/8/92 29/9/92 29/11/92
2/9/92

Ch 2: Square palindromes

15-16 marks = 2 stars
12-14 marks = 1 star

- Task 1:** 1, 121, 12321, 1234321 (4 marks)
123454321, 12345654321, 1234567654321, 123456787654321 (8 marks)
- Task 2:** 63, 7623, 776223,777776222223 (1,1,3 marks)

Ch 3: Square headscratchers

3 correct = 2 stars
2 correct = 1 star

1. 98
2. 676
3. $n = 6$

20 marks = 4 stars
17-19 marks = 3 stars
11-16 marks = 2 stars
9-10 marks = 1 star

Ch 4: Palindromic chains

Q4.	Number	Number of steps	Palindrome	Number	Number of steps	Palindrome
	11	1	22	31	1	44
	12	1	33	32	1	55
	13	1	44	33	1	66
	14	1	55	34	1	77
	15	1	66	35	1	88
	16	1	77	36	1	99
	17	1	88	37	2	121
	18	1	99	38	1	121
	19	2	121	39	2	363
	20	1	22	40	1	44
	21	1	33	41	1	55
	22	1	44	42	1	66
	23	1	55	43	1	77
	24	1	66	44	1	88
	25	1	77	45	1	99
	26	1	88	46	2	121
	27	1	99	47	1	121
	28	2	121	48	2	363
	29	1	121	49	2	484
	30	1	33			

5. 11 (1 mark)
6. digit sums = 12 (1 mark)
7. 39 & 48; 57 & 66; digit sums = 13 (4 marks)
8. 49 58 67 76 85 94 (6 marks)
9. 56,68,77,86,95 (DS = 14) reach 1111 in 3 stages.
69,78,87,96 (DS = 15) reach 4884 in 4 stages.
79,88,97 (DS = 16) reach 44044 in 4 stages.
89, 98 (DS = 18) do not reach a palindrome within the range of a calculator.
99 (DS=18) reaches 79497 in 6 stages. (8 marks)

NOTE:
YOU WILL NEED TO
EXTEND THE TABLE.

Ch 5: Digit sums

All correct = 1 star

- 57, 93, 369, 924, 7773
- 63, 198, 279, 405
- 36, 372
- Multiples of 3 (ie final digit sum = 3, 6 or 9) that are even.

Ch 6: Multiples of what ?

All correct = 1 star

- Task 1: $k = 9$ Task 2: $p = 11$

Ch 7: 37 is three times magic

All correct = 1 star

$37 \times 3 = 111$ $37 \times 6 = 222$ $37 \times 9 = 333$ $37 \times 12 = 444$ $37 \times 15 = 555$

Predict and check : $37 \times 27 = 999$

Connection: the sum of the digits of the answer is the number that 37 was multiplied by.

Ch 8: Abundant, perfect and deficient numbers

36 marks = 2 stars
30-35 marks = 1 star

- 2 marks for showing that 12 is abundant
- 2 marks for showing that 15 is deficient
- 2 marks for saying that the only proper factor of 3 is 1.

4.

Abundant numbers	: 12, 18, 20
Perfect numbers	: 6
Deficient numbers	: 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 19

 (20 marks)

- 6 and 28 (4 marks)
- 12, 18, 20, 24, 30, 36 (6 marks)

Ch 9: Multiple headscratcher

3 correct = 2 stars
2 correct = 1 star

- $N = 18$
- & 3. There are 3 sets of numbers with this property. Students are asked to find 2.
The sets are $[1, 4, 8 \Rightarrow 148, 481, 814]$ $[2, 5, 9 \Rightarrow 259, 592, 925]$
 $[2, 6, 9 \Rightarrow 296, 692, 962]$

Ch 10: Sum-square primes

10 correct = 2 stars
8-9 correct = 1 star

$13 = 9 + 4$ $17 = 1 + 16$ $29 = 25 + 4$ $37 = 36 + 1$ $41 = 25 + 16$
 $53 = 49 + 4$ $61 = 36 + 25$ $73 = 64 + 9$ $89 = 64 + 25$ $97 = 81 + 16$

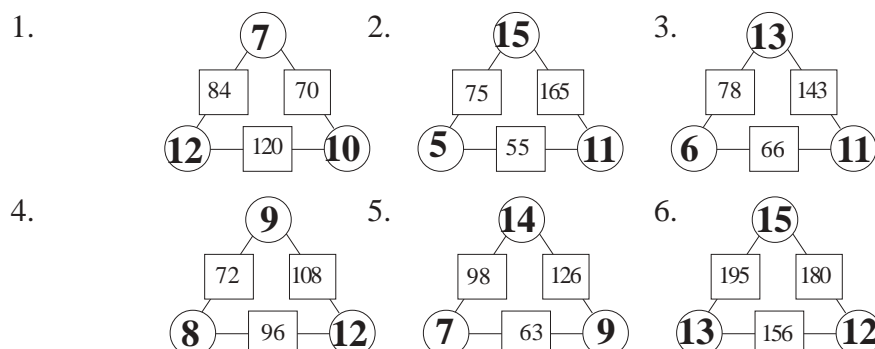
Ch 11: Sum-prime squares

17 marks = 2 stars
15-16 marks = 1 star

- | | | | |
|------------------------------|--|---------------------------|---------------|
| $4 = 2 + 2$ | $9 = 2 + 7$ | $16 = 11 + 5$ or $13 + 3$ | $25 = 23 + 2$ |
| $36 = 31 + 5$ or $19 + 17$ | $39 = 47 + 2$ | $64 = 61 + 3$ or $59 + 5$ | |
| $81 = 79 + 2$ | $100 = 89 + 11$ or $97 + 3$ | $121 = 119 + 2$ | |
| $144 = 139 + 5$ or $141 + 3$ | [only one way has been asked for, for each square] | | [11 marks] |
- The sums for the odd squares are all $k + 2$ (and they can only be done this one way) [1 mark]
- | | | |
|-----------------------------|------------------------------|---------------------------|
| $16 = 11 + 5$ or $13 + 3$ | $36 = 31 + 5$ or $19 + 17$ | $64 = 61 + 3$ or $59 + 5$ |
| $100 = 89 + 11$ or $97 + 3$ | $144 = 139 + 5$ or $141 + 3$ | [5 marks] |

Ch 12: Product arithmogon challenge

18 correct = 2 stars
15-17 correct = 1 star



Ch 13: Prime calculator search

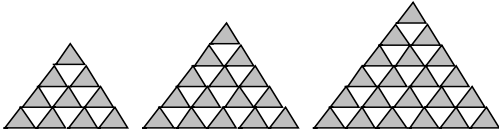
3 correct = 2 stars
2 correct = 1 star

1. $67 \times 73 = 4891$ 2. $71 \times 97 = 6887$ 3. $17 \times 29 \times 43 = 21199$

Ch 14: Triangular match patterns

12 marks = 2 stars
11 marks = 1 star

1. [3 marks]



2.

	Δ No 1	Δ No 2	Δ No 3	Δ No 4	Δ No 5	Δ No 6
shaded Δ s	1	3	6	10	15	21
unshaded Δ s	0	1	3	6	10	15
total Δ s	1	4	9	16	25	36
no. of matches	3	9	18	30	45	63

(4 marks if correct
but -1 mark for
each error)

3. Numbers in first and second rows are triangle numbers.
Numbers in third row are square numbers.
Fourth row is 3 x row 1
4. Row 2 is row 1 with each number moved one place to the right. (1 mark)